

K Voortgezette integraalrekening

Voorkennis Afgeleiden en primitieven

Bladzijde 181

- 1** a $f(x) = \sqrt{6x+1}$ geeft $f'(x) = \frac{1}{2\sqrt{6x+1}} \cdot 6 = \frac{3}{\sqrt{6x+1}}$
- b $f(x) = \frac{4}{\sqrt{2x-1}} = 4(2x-1)^{-\frac{1}{2}}$ geeft $f'(x) = -2(2x-1)^{-\frac{1}{2}} \cdot 2 = -\frac{4}{(2x-1)\sqrt{2x-1}}$
- c $f(x) = \ln(x^2+2x)$ geeft $f'(x) = \frac{1}{x^2+2x} \cdot (2x+2) = \frac{2x+2}{x^2+2x}$
- d $f(x) = 2^{4x-1}$ geeft $f'(x) = 2^{4x-1} \cdot \ln(2) \cdot 4 = 4 \ln(2) \cdot 2^{4x-1}$
- e $f(x) = \sin(x^2-x)$ geeft $f'(x) = \cos(x^2-x) \cdot (2x-1) = (2x-1)\cos(x^2-x)$
- f $f(x) = \tan(4x - \frac{1}{3}\pi)$ geeft $f'(x) = \frac{1}{\cos^2(4x - \frac{1}{3}\pi)} \cdot 4 = \frac{4}{\cos^2(4x - \frac{1}{3}\pi)}$
- 2** a $f(x) = e^x \sin(2x)$ geeft $f'(x) = e^x \sin(2x) + e^x \cdot 2 \cos(2x) = (\sin(2x) + 2 \cos(2x))e^x$
- b $f(x) = \frac{\ln(x)}{x^2+1}$ geeft $f'(x) = \frac{(x^2+1) \cdot \frac{1}{x} - \ln(x) \cdot 2x}{(x^2+1)^2} = \frac{x^2+1-2x^2 \ln(x)}{x(x^2+1)^2}$
- c $f(x) = \frac{1}{2}x^2 \ln(x)$ geeft $f'(x) = x \cdot \ln(x) + \frac{1}{2}x^2 \cdot \frac{1}{x} = x \ln(x) + \frac{1}{2}x$
- d $f(x) = \frac{2^x+1}{2^x} = 1 + 2^{-x}$ geeft $f'(x) = 2^{-x} \cdot \ln(2) \cdot -1 = -\ln(2) \cdot \frac{1}{2^x} = -\frac{\ln(2)}{2^x}$
- e $f(x) = 2x \tan(x)$ geeft $f'(x) = 2 \cdot \tan(x) + 2x \cdot \frac{1}{\cos^2(x)} = 2 \tan(x) + \frac{2x}{\cos^2(x)}$
- f $f(x) = \frac{4x+2}{\sqrt{2x+1}} = \frac{2(2x+1)}{\sqrt{2x+1}} = 2\sqrt{2x+1}$ geeft $f'(x) = 2 \cdot \frac{1}{2\sqrt{2x+1}} \cdot 2 = \frac{2}{\sqrt{2x+1}}$

Bladzijde 182

- 3** a $f(x) = x^2 + \sin(x)$ geeft $F(x) = \frac{1}{3}x^3 - \cos(x) + c$
- b $f(x) = \frac{x^2-2}{x^3} = \frac{1}{x} - 2x^{-3}$ geeft $F(x) = \ln|x| + x^{-2} + c = \ln|x| + \frac{1}{x^2} + c$
- c $f(x) = 4 \cdot 3^x$ geeft $F(x) = 4 \cdot \frac{3^x}{\ln(3)} + c = \frac{4 \cdot 3^x}{\ln(3)} + c$
- d $f(x) = \ln\left(\frac{2}{\sqrt{x}}\right) = \ln(2) - \ln(x^{\frac{1}{2}}) = \ln(2) - \frac{1}{2}\ln(x)$ geeft
 $F(x) = x \ln(2) - \frac{1}{2}(x \ln(x) - x) + c = x \ln(2) - \frac{1}{2}x \ln(x) + \frac{1}{2}x + c$
- e $f(x) = 6 \cdot \log(3x)$ geeft $F(x) = 6 \cdot \frac{1}{\ln(10)} (3x \ln(3x) - 3x) + c = \frac{6x(\ln(3x) - 1)}{\ln(10)} + c$
- f $f(x) = \frac{4^x - 2^x + 1}{2^x} = 2^x - 1 + 2^{-x}$ geeft $F(x) = \frac{2^x}{\ln(2)} - x - \frac{2^{-x}}{\ln(2)} + c = \frac{2^x}{\ln(2)} - x - \frac{1}{2^x \ln(2)} + c$
- 4** a $f(x) = e^{\frac{1}{2}x-1}$ geeft $F(x) = 2e^{\frac{1}{2}x-1} + c$
- b $f(x) = \frac{3}{2x-1}$ geeft $F(x) = 3 \cdot \frac{1}{2} \ln|2x-1| + c = \frac{3}{2} \ln|2x-1| + c$
- c $f(x) = 4 \sin(\pi x)$ geeft $F(x) = 4 \cdot \frac{1}{\pi} \cdot -\cos(\pi x) + c = -\frac{4 \cos(\pi x)}{\pi} + c$
- d $f(x) = \frac{4x+1}{2\sqrt{4x+1}} = \frac{1}{2} \sqrt{4x+1} = \frac{1}{2}(4x+1)^{\frac{1}{2}}$ geeft
 $F(x) = \frac{1}{2} \cdot \frac{1}{\frac{1}{2}} \cdot \frac{2}{3} (4x+1)^{\frac{3}{2}} + c = \frac{1}{12} (4x+1) \sqrt{4x+1} + c$
- e $f(x) = (\frac{1}{2})^{2x-3}$ geeft $F(x) = \frac{1}{2} \cdot \frac{(\frac{1}{2})^{2x-3}}{\ln(\frac{1}{2})} + c = \frac{\frac{1}{2} \cdot (\frac{1}{2})^{2x-3}}{\ln(\frac{1}{2})} + c = \frac{(\frac{1}{2})^{2x-2}}{\ln(\frac{1}{2})} + c$
- f $f(x) = 4 \ln(x-1)$ geeft $F(x) = 4((x-1) \ln(x-1) - (x-1)) + c = 4(x-1) \ln(x-1) - 4(x-1) + c$

- 5** a $\int_{\frac{5}{6}\pi}^{\frac{1}{2}\pi} \sin(2x) dx = \left[-\frac{1}{2} \cos(2x)\right]_{\frac{5}{6}\pi}^{\frac{1}{2}\pi} = -\frac{1}{2} \cos\left(\frac{2}{3}\pi\right) + \frac{1}{2} \cos\left(\frac{1}{3}\pi\right) = -\frac{1}{2} \cdot -\frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$
- b $\int_1^4 \frac{6}{2x-1} dx = \left[6 \cdot \frac{1}{2} \ln|2x-1|\right]_1^4 = 3 \ln(7) - 3 \ln(1) = 3 \ln(7)$
- c $\int_e^{e^2} 10 \ln(\sqrt[4]{x}) dx = \int_e^{e^2} 10 \ln(x^{\frac{1}{4}}) dx = \int_e^{e^2} 2\frac{1}{2} \ln(x) dx = \left[2\frac{1}{2}(x \ln(x) - x)\right]_e^{e^2}$
 $= 2\frac{1}{2}(e^2 \ln(e^2) - e^2 - (e \ln(e) - e)) = 2\frac{1}{2}(e^2 \cdot 2 - e^2 - (e \cdot 1 - e)) = 2\frac{1}{2}e^2$
- d $\int_0^4 6e^{\frac{1}{2}x-3} dx = \left[6 \cdot 2e^{\frac{1}{2}x-3}\right]_0^4 = 12e^{-1} - 12e^{-3} = \frac{12}{e} - \frac{12}{e^3}$

K.1 De substitutiemethode

Bladzijde 183

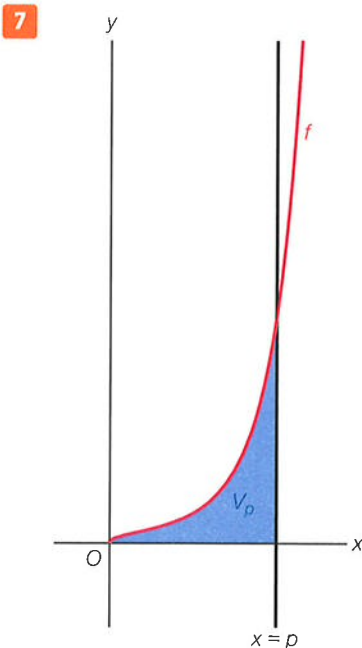
- 1** a $f(x) = \sin(x^2 + x)$ geeft $f'(x) = \cos(x^2 + x) \cdot (2x + 1) = (2x + 1) \cos(x^2 + x)$
 b $G'(x) = (2x + 1) \cos(x^2 + x)$, dus $G(x) = \sin(x^2 + x) + 3$ is een primitieve van $g(x) = (2x + 1) \cos(x^2 + x)$.

Bladzijde 184

- 2** a Omdat $\frac{d(x^2 + 1)}{dx} = 2x$ en $6x = 3 \cdot 2x$ lukt het primitiveren van $f(x)$ wel op deze manier.
 Omdat $\frac{d(x^3 + 1)}{dx} = 3x^2$ en $6x$ niet geschreven kan worden als het product van een constante en $3x^2$ lukt het primitiveren van $h(x)$ niet op deze manier.
- b Omdat $\frac{d(x^4 + 7)}{dx} = 4x^3$ en $10x^3 = 2\frac{1}{2} \cdot 4x^3$ lukt het primitiveren van $g(x)$ wel op deze manier.
 Omdat $\frac{d(x^4 + x)}{dx} = 4x^3 + 1$ en $10x^3$ niet geschreven kan worden als het product van een constante en $4x^3 + 1$ lukt het primitiveren van $k(x)$ niet op deze manier.
- c $\frac{d(x^4 + x)}{dx} = 4x^3 + 1$
 $10x^3 + a = 2\frac{1}{2}(4x^3 + \frac{2}{5}a)$ moet gelijk zijn aan $2\frac{1}{2}(4x^3 + 1)$, dus $\frac{2}{5}a = 1$ en dit geeft $a = 2\frac{1}{2}$.
- 3** a $3x^2 dx = d(x^3 + 5)$
 b $5x^4 dx = d(x^5 - 3)$
 c $\cos(2x) dx = d(\frac{1}{2} \sin(2x) + \pi)$
 d $\frac{1}{x} dx = d \ln(x)$
 e $(5 - 2x) dx = d(5x - x^2 + 5)$
 f $2 \cos(4x) dx = d\frac{1}{2} \sin(4x)$
- 4** a $F(x) = \int 2x(x^2 + 4)^3 dx = \int (x^2 + 4)^3 \cdot 2x dx = \int (x^2 + 4)^3 d(x^2 + 4) = \int u^3 du = \frac{1}{4} u^4 + c$
 $= \frac{1}{4} (x^2 + 4)^4 + c$
 b $G(x) = \int 6x\sqrt{x^2 + 1} dx = \int 3\sqrt{x^2 + 1} \cdot 2x dx = \int 3\sqrt{x^2 + 1} d(x^2 + 1) = \int 3u^{\frac{1}{2}} du = \frac{3}{\frac{1}{2} + 1} u^{\frac{1}{2} + 1} + c$
 $= 2u\sqrt{u} + c = 2(x^2 + 1)\sqrt{x^2 + 1} + c$
 c $H(x) = \int 6x^2(x^3 - 1)^4 dx = \int 2(x^3 - 1)^4 \cdot 3x^2 dx = \int 2(x^3 - 1)^4 d(x^3 - 1) = \int 2u^4 du = \frac{2}{5} u^5 + c$
 $= \frac{2}{5} (x^3 - 1)^5 + c$
 d $J(x) = \int 3x^2 \sin(x^3 - 1) dx = \int \sin(x^3 - 1) \cdot 3x^2 dx = \int \sin(x^3 - 1) d(x^3 - 1) = \int \sin(u) du$
 $= -\cos(u) + c = -\cos(x^3 - 1) + c$

Bladzijde 185

- 5 a** $F(x) = \int x(3x^2 - 4)^3 dx = \int \frac{1}{6}(3x^2 - 4)^3 \cdot 6x dx = \int \frac{1}{6}(3x^2 - 4)^3 d(3x^2 - 4) = \int \frac{1}{6}u^3 du = \frac{1}{24}u^4 + c$
 $= \frac{1}{24}(3x^2 - 4)^4 + c$
- b** $F(x) = \int 2x\sqrt{x^2 - 3} dx = \int \sqrt{x^2 - 3} d(x^2 - 3) = \int \sqrt{u} du = \frac{2}{3}u^{1/2} + c = \frac{2}{3}(x^2 - 3)\sqrt{x^2 - 3} + c$
- c** $F(x) = \int \frac{6x}{3x^2 + 2} dx = \int \frac{1}{3x^2 + 2} \cdot 6x dx = \int \frac{1}{3x^2 + 2} d(3x^2 + 2) = \int \frac{1}{u} du = \ln|u| + c = \ln|3x^2 + 2| + c = \ln(3x^2 + 2) + c$
- d** $F(x) = \int x \ln(x^2 + 1) dx = \int \frac{1}{2} \ln(x^2 + 1) \cdot 2x dx = \int \frac{1}{2} \ln(x^2 + 1) d(x^2 + 1) = \int \frac{1}{2} \ln(u) du$
 $= \frac{1}{2}(u \ln(u) - u) + c = \frac{1}{2}((x^2 + 1) \ln(x^2 + 1) - (x^2 + 1)) + c$
- 6 a** $F(x) = \int \frac{\ln(x)}{x} dx = \int \ln(x) \cdot \frac{1}{x} dx = \int \ln(x) d \ln(x) = \int u du = \frac{1}{2}u^2 + c = \frac{1}{2} \ln^2(x) + c$
- b** $G(x) = \int x e^{-x^2} dx = \int -\frac{1}{2} e^{-x^2} \cdot -2x dx = \int -\frac{1}{2} e^{-x^2} d(-x^2) = \int -\frac{1}{2} e^u du = -\frac{1}{2} e^u + c = -\frac{1}{2} e^{-x^2} + c$
- c** $H(x) = \int x \sqrt{5 - x^2} dx = \int -\frac{1}{2} \sqrt{5 - x^2} \cdot -2x dx = \int -\frac{1}{2} \sqrt{5 - x^2} d(5 - x^2) = \int -\frac{1}{2} u^{1/2} du$
 $= -\frac{1}{2} \cdot \frac{1}{1/2} u^{1/2} + c = -\frac{1}{3} u^{3/2} + c = -\frac{1}{3} (5 - x^2) \sqrt{5 - x^2} + c$
- d** $J(x) = \int \frac{x}{\sqrt{x^2 + 1}} dx = \int \frac{1}{2} \cdot \frac{1}{\sqrt{x^2 + 1}} \cdot 2x dx = \int \frac{1}{2\sqrt{x^2 + 1}} d(x^2 + 1) = \int \frac{1}{2\sqrt{u}} du = \sqrt{u} + c = \sqrt{x^2 + 1} + c$



$$\int (f(x))^2 dx = \int \frac{1}{100} \cdot 3x \cdot (e^{x^2})^2 dx = \int \frac{3}{400} \cdot e^{2x^2} \cdot 4x dx = \int \frac{3}{400} \cdot e^{2x^2} d(2x^2) = \int \frac{3}{400} \cdot e^u du = \frac{3}{400} e^u = \frac{3}{400} e^{2x^2}$$

$$I(L_p) = \pi \int_0^p (f(x))^2 dx = \pi \left[\frac{3}{400} e^{2x^2} \right]_0^p = \pi \left(\frac{3}{400} \cdot e^{2p^2} - \frac{3}{400} \cdot 1 \right) = \frac{3}{400} \pi (e^{2p^2} - 1)$$

$$I(L_p) = 3 \frac{3}{10} \pi \text{ geeft } \frac{3}{400} \pi (e^{2p^2} - 1) = 3 \frac{3}{10} \pi$$

$$e^{2p^2} - 1 = 440$$

$$e^{2p^2} = 441$$

$$2p^2 = \ln(441)$$

$$p^2 = \frac{1}{2} \ln(441)$$

$$p^2 = \ln(\sqrt{441})$$

$$p^2 = \ln(21)$$

$$p = \sqrt{\ln(21)}$$

8 a $f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{1}{\cos(x)} \cdot \sin(x)$

b $\int \tan(x) dx = \int \frac{\sin(x)}{\cos(x)} dx = \int \frac{-1}{\cos(x)} \cdot -\sin(x) dx = \int \frac{-1}{\cos(x)} d\cos(x)$

Bladzijde 186

9 a $\int_0^{\frac{1}{2}\pi} \tan(2x) dx = \left[-\frac{1}{2} \ln|\cos(2x)| \right]_0^{\frac{1}{2}\pi} = -\frac{1}{2} \ln|\cos(\frac{1}{2}\pi)| + \frac{1}{2} \ln|\cos(0)| = -\frac{1}{2} \ln(\frac{1}{2}) + \frac{1}{2} \ln(1)$
 $= -\frac{1}{2} \ln(2^{-1}) + \frac{1}{2} \cdot 0 = \frac{1}{2} \ln(2)$

b $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \sin^3(x) dx = \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \sin^2(x) \cdot \sin(x) dx = \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} -(1 - \cos^2(x)) \cdot -\sin(x) dx = \int_{x=\frac{1}{4}\pi}^{x=\frac{1}{2}\pi} -(1 - \cos^2(x)) d\cos(x)$
 $= \int_{\frac{1}{2}\sqrt{2}}^{\frac{1}{2}} (-1 + u^2) du = \left[-u + \frac{1}{3}u^3 \right]_{\frac{1}{2}\sqrt{2}}^{\frac{1}{2}} = -\frac{1}{2} + \frac{1}{3} \cdot \left(\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\sqrt{2} + \frac{1}{3} \cdot \left(\frac{1}{2}\sqrt{2}\right)^3\right)$
 $= -\frac{1}{2} + \frac{1}{24} + \frac{1}{2}\sqrt{2} - \frac{1}{12}\sqrt{2} = -\frac{11}{24} + \frac{5}{12}\sqrt{2}$

c $\int_0^{\frac{1}{2}\pi} \frac{\sin(x)}{\cos^3(x)} dx = \int_0^{\frac{1}{2}\pi} -\frac{1}{\cos^3(x)} \cdot -\sin(x) dx = \int_{x=0}^{x=\frac{1}{2}\pi} -\frac{1}{\cos^3(x)} d\cos(x) = \int_1^{\frac{1}{2}\sqrt{3}} -\frac{1}{u^3} du = \int_1^{\frac{1}{2}\sqrt{3}} -u^{-3} du$
 $= \left[\frac{1}{2}u^{-2} \right]_1^{\frac{1}{2}\sqrt{3}} = \left[\frac{1}{2u^2} \right]_1^{\frac{1}{2}\sqrt{3}} = \frac{1}{2 \cdot \frac{3}{4}} - \frac{1}{2 \cdot 1} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

Bladzijde 187

10 a $\int \frac{\cos(x)}{\sin^2(x)} dx = \int \frac{1}{\sin^2(x)} \cdot \cos(x) dx = \int \frac{1}{\sin^2(x)} d\sin(x) = \int \frac{1}{u^2} du = \int u^{-2} du = -u^{-1} + c = -\frac{1}{u} + c$
 $= -\frac{1}{\sin(x)} + c$

$f(x) = \cos(x) - \frac{\cos(x)}{\sin^2(x)}$ geeft $F(x) = \sin(x) + \frac{1}{\sin(x)} + c$

b $G(x) = \int \sin^5(x) dx = \int -\sin^4(x) \cdot -\sin(x) dx = \int -(\sin^2(x))^2 \cdot -\sin(x) dx = \int -(1 - \cos^2(x))^2 d\cos(x)$
 $= \int -(1 - u^2)^2 du = \int -(1 - 2u^2 + u^4) du = \int (-1 + 2u^2 - u^4) du = -u + \frac{2}{3}u^3 - \frac{1}{5}u^5 + c$
 $= -\frac{1}{5}u^5 + \frac{2}{3}u^3 - u + c = -\frac{1}{5}\cos^5(x) + \frac{2}{3}\cos^3(x) - \cos(x) + c$

11 a $\int_1^2 \frac{\ln^2(x)}{x} dx = \int_1^2 \ln^2(x) \cdot \frac{1}{x} dx = \int_{x=1}^{x=2} \ln^2(x) d\ln(x) = \int_0^{\ln(2)} u^2 du = \left[\frac{1}{3}u^3 \right]_0^{\ln(2)} = \frac{1}{3}\ln^3(2)$

b $\int_e^{e^2} \frac{1}{x \ln(x)} dx = \int_e^{e^2} \frac{1}{\ln(x)} \cdot \frac{1}{x} dx = \int_{x=e}^{x=e^2} \frac{1}{\ln(x)} d\ln(x) = \int_1^2 \frac{1}{u} du = [\ln|u|]_1^2 = \ln(2) - \ln(1) = \ln(2)$

c $\int_0^1 \frac{x^2}{e^{2x^3}} dx = \int_0^1 -\frac{1}{6} \cdot e^{-2x^3} \cdot -6x^2 dx = \int_{x=0}^{x=1} -\frac{1}{6} \cdot e^{-2x^3} d(-2x^3) = \int_0^{-2} -\frac{1}{6} e^u du = \left[-\frac{1}{6} e^u \right]_0^{-2} = -\frac{1}{6} e^{-2} + \frac{1}{6} e^0$
 $= -\frac{1}{6e^2} + \frac{1}{6} = \frac{e^2 - 1}{6e^2}$

12 a $x = 0$ geeft $u = \sqrt{0+4} = 2$
 $x = 5$ geeft $u = \sqrt{5+4} = 3$

b $\sqrt{x+4} = u$

kwadrateren geeft

$x + 4 = u^2$

$x = u^2 - 4$

Omdat $x = u^2 - 4$ en $\sqrt{x+4} = u$ krijg je $\int x\sqrt{x+4} dx = \int (u^2 - 4) \cdot u d(u^2 - 4)$.

c $\int (u^2 - 4) \cdot u d(u^2 - 4) = \int (u^2 - 4) \cdot u \cdot 2u du = \int (2u^4 - 8u^2) du$

$$\begin{aligned} \text{d} \int_0^5 x\sqrt{x+4} dx &= \int_2^3 (2u^4 - 8u^2) du = \left[\frac{2}{5}u^5 - \frac{8}{3}u^3 \right]_2^3 = \frac{2}{5} \cdot 3^5 - \frac{8}{3} \cdot 3^3 - \frac{2}{5} \cdot 2^5 + \frac{8}{3} \cdot 2^3 \\ &= 97\frac{1}{5} - 72 - 12\frac{4}{5} + 21\frac{1}{3} = 33\frac{11}{15} \end{aligned}$$

13 a $\sqrt{x+1} = u$
kwadrateren geeft
 $x+1 = u^2$
 $x = u^2 - 1$

$$\begin{aligned} \int_0^3 x\sqrt{x+1} dx &= \int_{u=1}^{u=2} (u^2 - 1) \cdot u du (u^2 - 1) = \int_1^2 (u^2 - 1) \cdot u \cdot 2u du = \int_1^2 (2u^4 - 2u^2) du = \left[\frac{2}{5}u^5 - \frac{2}{3}u^3 \right]_1^2 \\ &= \frac{2}{5} \cdot 2^5 - \frac{2}{3} \cdot 2^3 - \left(\frac{2}{5} \cdot 1^5 - \frac{2}{3} \cdot 1^3 \right) = 12\frac{4}{5} - 5\frac{1}{3} - \frac{2}{5} + \frac{2}{3} = 7\frac{11}{15} \end{aligned}$$

b $\sqrt{x+9} = u$
kwadrateren geeft
 $x+9 = u^2$
 $x = u^2 - 9$

$$\begin{aligned} \int_0^7 \frac{5x}{\sqrt{x+9}} dx &= \int_{u=3}^{u=4} \frac{5(u^2 - 9)}{u} d(u^2 - 9) = \int_3^4 \frac{5(u^2 - 9)}{u} \cdot 2u du = \int_3^4 (10u^2 - 90) du = \left[\frac{10}{3}u^3 - 90u \right]_3^4 \\ &= \frac{10}{3} \cdot 4^3 - 90 \cdot 4 - \left(\frac{10}{3} \cdot 3^3 - 90 \cdot 3 \right) = 213\frac{1}{3} - 360 - 90 + 270 = 33\frac{1}{3} \end{aligned}$$

14 a $f(x) = 0$ geeft $\frac{2 + \ln(x)}{x} = 0$
 $2 + \ln(x) = 0$
 $\ln(x) = -2$
 $x = e^{-2}$

$$\begin{aligned} O(V) &= \int_{e^{-2}}^e \frac{2 + \ln(x)}{x} dx = \int_{e^{-2}}^e (2 + \ln(x)) \cdot \frac{1}{x} dx = \int_{x=e^{-2}}^{x=e} (2 + \ln(x)) d \ln(x) = \int_{-2}^1 (2 + u) du \\ &= \left[2u + \frac{1}{2}u^2 \right]_{-2}^1 = 2 + \frac{1}{2} - (-4 + 2) = 4\frac{1}{2} \end{aligned}$$

b $\int_1^p \frac{2 + \ln(x)}{x} dx = \int_1^p (2 + \ln(x)) \cdot \frac{1}{x} dx = \int_{x=1}^{x=p} (2 + \ln(x)) d \ln(x) = \int_0^{\ln(p)} (2 + u) du = \left[2u + \frac{1}{2}u^2 \right]_0^{\ln(p)}$
 $= 2 \ln(p) + \frac{1}{2} \ln^2(p)$

$$\begin{aligned} \int_1^p f(x) dx &= 6 \text{ geeft } 2 \ln(p) + \frac{1}{2} \ln^2(p) = 6 \\ \ln^2(p) + 4 \ln(p) - 12 &= 0 \\ (\ln(p) - 2)(\ln(p) + 6) &= 0 \\ \ln(p) &= 2 \vee \ln(p) = -6 \\ p &= e^2 \vee p = e^{-6} \\ &\text{vold. niet} \end{aligned}$$

Dus $p = e^2$.

Bladzijde 188

15 a $f(x) = g(x)$ geeft $\frac{4 \ln^2(x)}{x} = \frac{1}{x}$
 $4 \ln^2(x) = 1$
 $\ln^2(x) = \frac{1}{4}$
 $\ln(x) = \frac{1}{2} \vee \ln(x) = -\frac{1}{2}$
 $x = e^{\frac{1}{2}} \vee x = e^{-\frac{1}{2}}$
 $x = \sqrt{e} \vee x = \frac{1}{\sqrt{e}}$

$$f(x) \leq g(x) \text{ geeft } \frac{1}{\sqrt{e}} \leq x \leq \sqrt{e}$$

$$\begin{aligned}
 \text{b } O(V) &= \int_{\frac{1}{\sqrt{e}}}^{\sqrt{e}} (g(x) - f(x)) dx = \int_{\frac{1}{\sqrt{e}}}^{\sqrt{e}} \left(\frac{1}{x} - \frac{4 \ln^2(x)}{x} \right) dx = \int_{\frac{1}{\sqrt{e}}}^{\sqrt{e}} (1 - 4 \ln^2(x)) \cdot \frac{1}{x} dx \\
 &= \int_{x=\frac{1}{\sqrt{e}}}^{x=\sqrt{e}} (1 - 4 \ln^2(x)) d \ln(x) = \int_{-\frac{1}{2}}^{\frac{1}{2}} (1 - 4u^2) du = \left[u - \frac{4}{3} u^3 \right]_{-\frac{1}{2}}^{\frac{1}{2}} \\
 &= \frac{1}{2} - \frac{4}{3} \cdot \frac{1}{8} - \left(-\frac{1}{2} + \frac{4}{3} \cdot -\frac{1}{8} \right) = \frac{1}{2} - \frac{1}{6} + \frac{1}{2} - \frac{1}{6} = \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{16 a } f(x) &= \frac{3x^2}{x^3 + 4} \text{ geeft } f'(x) = \frac{(x^3 + 4) \cdot 6x - 3x^2 \cdot 3x^2}{(x^3 + 4)^2} = \frac{6x^4 + 24x - 9x^4}{(x^3 + 4)^2} = \frac{-3x^4 + 24x}{(x^3 + 4)^2} \\
 f'(x) &= 0 \text{ geeft } -3x^4 + 24x = 0 \\
 &\quad -3x(x^3 - 8) = 0 \\
 &\quad x = 0 \vee x^3 = 8 \\
 &\quad x = 0 \vee x = 2
 \end{aligned}$$

$$x^3 + 4 = 0$$

$$x^3 = -4$$

$$x = \sqrt[3]{-4}$$

$$\text{min. is } f(0) = 0$$

$$\text{max. is } f(2) = 1$$

$f(x) = p$ heeft drie oplossingen voor $0 < p < 1$.

$$\begin{aligned}
 \text{b } \int_0^p \frac{3x^2}{x^3 + 4} dx &= \int_0^p \frac{1}{x^3 + 4} \cdot 3x^2 dx = \int_{x=0}^{x=p} \frac{1}{x^3 + 4} d(x^3 + 4) = \int_4^{p^3+4} \frac{1}{u} du = [\ln|u|]_4^{p^3+4} = \ln(p^3 + 4) - \ln(4) \\
 \int_0^p f(x) dx &= 2 \text{ geeft } \ln(p^3 + 4) - \ln(4) = 2 \\
 \ln\left(\frac{p^3 + 4}{4}\right) &= 2 \\
 \frac{p^3 + 4}{4} &= e^2 \\
 p^3 + 4 &= 4e^2 \\
 p^3 &= 4e^2 - 4 \\
 p &= \sqrt[3]{4e^2 - 4}
 \end{aligned}$$

K.2 Partieel integreren

Bladzijde 190

$$\text{17 a } k(x) = (2x + 3) \sin(x) \text{ differentiëren met de productregel geeft } k'(x) = 2 \sin(x) + (2x + 3) \cos(x). \text{ Dus is } k \text{ geen primitieve van } h.$$

$$\text{b } \text{Integreren van het linker- en rechterlid van de vergelijking } (2x + 3) \cos(x) = k'(x) - 2 \sin(x) \text{ geeft}$$

$$\int (2x + 3) \cos(x) dx = \int (k'(x) - 2 \sin(x)) dx$$

$$\int (2x + 3) \cos(x) dx = (2x + 3) \sin(x) - \int 2 \sin(x) dx.$$

$$\text{c } H(x) = \int (2x + 3) \cos(x) dx = (2x + 3) \sin(x) - \int 2 \sin(x) dx = (2x + 3) \sin(x) + 2 \cos(x) + c$$

Bladzijde 192

$$\begin{aligned}
 \text{18 a } \int \frac{\ln(x)}{\sqrt{x}} dx &= \int \ln(x) \cdot x^{-\frac{1}{2}} dx = \int \ln(x) d 2\sqrt{x} = \ln(x) \cdot 2\sqrt{x} - \int 2\sqrt{x} d \ln(x) \\
 &= 2\sqrt{x} \cdot \ln(x) - \int 2\sqrt{x} \cdot \frac{1}{x} dx = 2\sqrt{x} \cdot \ln(x) - \int 2x^{-\frac{1}{2}} dx \\
 &= 2\sqrt{x} \cdot \ln(x) - 4x^{\frac{1}{2}} + c = 2\sqrt{x} (\ln(x) - 2) + c
 \end{aligned}$$

$$\text{b } \int x \sin(x) dx = \int \sin(x) d \frac{1}{2} x^2 = \frac{1}{2} x^2 \sin(x) - \int \frac{1}{2} x^2 \cos(x) dx = \dots$$

Het wordt op deze manier alleen maar ingewikkelder, dus het lukt zo niet.

$$\text{c } F(x) = \int \ln(x) dx = x \ln(x) - \int x d \ln(x) = x \ln(x) - \int x \cdot \frac{1}{x} dx = x \ln(x) - \int 1 dx = x \ln(x) - x + c$$

- 19 a** $F(x) = \int x e^{2x} dx = \int x d\frac{1}{2}e^{2x} = x \cdot \frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x} dx = \frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x} + c$
- b** $F(x) = \int 2x \cos(x) dx = \int 2x d \sin(x) = 2x \sin(x) - \int \sin(x) d 2x = 2x \sin(x) - \int 2 \sin(x) dx$
 $= 2x \sin(x) + 2 \cos(x) + c$
- c** $F(x) = \int x \ln(x) dx = \int \ln(x) d\frac{1}{2}x^2 = \frac{1}{2}x^2 \ln(x) - \int \frac{1}{2}x^2 d \ln(x) = \frac{1}{2}x^2 \ln(x) - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx$
 $= \frac{1}{2}x^2 \ln(x) - \int \frac{1}{2}x dx = \frac{1}{2}x^2 \ln(x) - \frac{1}{4}x^2 + c$
- d** $\int x^3 \ln(x) dx = \int \ln(x) d\frac{1}{4}x^4 = \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^4 d \ln(x) = \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^4 \cdot \frac{1}{x} dx$
 $= \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^3 dx = \frac{1}{4}x^4 \ln(x) - \frac{1}{16}x^4 + c$
Dus $f(x) = x^3 \ln(x) + 3$ geeft $F(x) = \frac{1}{4}x^4 \ln(x) - \frac{1}{16}x^4 + 3x + c$
- 20 a** $\int_0^1 2x e^{x+1} dx = \int 2x d e^{x+1} = 2x e^{x+1} - \int e^{x+1} d 2x = 2x e^{x+1} - \int e^{x+1} \cdot 2 dx = 2x e^{x+1} - 2 e^{x+1} + c$
 $\int_0^1 2x e^{x+1} dx = [2x e^{x+1} - 2 e^{x+1}]_0^1 = 2e^2 - 2e^2 - (0 - 2e) = 2e$
- b** $\int (3x+1) \sin(x) dx = \int -(3x+1) d \cos(x) = -(3x+1) \cdot \cos(x) - \int -\cos(x) d(3x+1)$
 $= -(3x+1) \cos(x) + \int 3 \cos(x) dx = -(3x+1) \cos(x) + 3 \sin(x) + c$
 $\int_0^\pi (3x+1) \sin(x) dx = [-(3x+1) \cos(x) + 3 \sin(x)]_0^\pi$
 $= -(3\pi+1) \cos(\pi) + 3 \sin(\pi) - (-\cos(0) + 3 \sin(0)) = 3\pi + 1 + 1 = 3\pi + 2$

- 21 a** $f(x) = x^2 \ln(x)$ geeft $f'(x) = 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x} = 2x \ln(x) + x$

$$f'(x) = 0 \text{ geeft } 2x \ln(x) + x = 0$$

$$x(2 \ln(x) + 1) = 0$$

$$x = 0 \vee 2 \ln(x) + 1 = 0$$

$$x = 0 \vee \ln(x) = -\frac{1}{2}$$

$$x = 0 \vee x = e^{-\frac{1}{2}}$$

$$\text{vold. niet}$$

$$f(e^{-\frac{1}{2}}) = (e^{-\frac{1}{2}})^2 \cdot -\frac{1}{2} = -\frac{1}{2}e^{-1}$$

$$\text{Dus } A(e^{-\frac{1}{2}}, -\frac{1}{2}e^{-1}) = A\left(\frac{1}{\sqrt{e}}, -\frac{1}{2e}\right).$$

- b** $\int x^2 \ln(x) dx = \int \ln(x) d\frac{1}{3}x^3 = \frac{1}{3}x^3 \ln(x) - \int \frac{1}{3}x^3 d \ln(x) = \frac{1}{3}x^3 \ln(x) - \int \frac{1}{3}x^3 \cdot \frac{1}{x} dx$
 $= \frac{1}{3}x^3 \ln(x) - \int \frac{1}{3}x^2 dx = \frac{1}{3}x^3 \ln(x) - \frac{1}{9}x^3 + c$
 $O(V) = \int_1^e x^2 \ln(x) dx = \left[\frac{1}{3}x^3 \ln(x) - \frac{1}{9}x^3\right]_1^e = \frac{1}{3}e^3 \ln(e) - \frac{1}{9}e^3 - \left(\frac{1}{3} \ln(1) - \frac{1}{9}\right) = \frac{2}{9}e^3 + \frac{1}{9}$

Bladzijde 193

- 22 a** $f(x) = \frac{\ln(x)}{x^2}$ geeft $f'(x) = \frac{x^2 \cdot \frac{1}{x} - \ln(x) \cdot 2x}{x^4} = \frac{x - 2x \ln(x)}{x^4} = \frac{1 - 2 \ln(x)}{x^3}$

$$f'(x) = 0 \text{ geeft } 1 - 2 \ln(x) = 0$$

$$\ln(x) = \frac{1}{2}$$

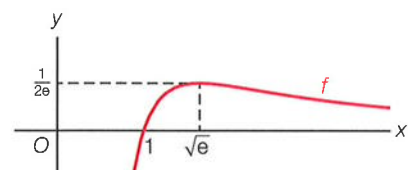
$$x = e^{\frac{1}{2}}$$

$$x = \sqrt{e}$$

Zie de figuur hiernaast.

$$\text{max. is } f(\sqrt{e}) = \frac{\frac{1}{2}}{(\sqrt{e})^2} = \frac{\frac{1}{2}}{e} = \frac{1}{2e}$$

$$\text{Dus } B_f = \left\langle \leftarrow, \frac{1}{2e} \right\rangle.$$



$$\begin{aligned}
 \text{b } \int \frac{\ln(x)}{x^2} dx &= \int \ln(x) \cdot x^{-2} dx = \int \ln(x) d(-x^{-1}) = -x^{-1} \ln(x) - \int -x^{-1} d \ln(x) = -\frac{1}{x} \ln(x) + \int \frac{1}{x} \cdot \frac{1}{x} dx \\
 &= -\frac{\ln(x)}{x} + \int x^{-2} dx = -\frac{\ln(x)}{x} - x^{-1} + c = -\frac{\ln(x) + 1}{x} + c \\
 O(V) &= \int_1^e \frac{\ln(x)}{x^2} dx = \left[-\frac{\ln(x) + 1}{x} \right]_1^e = -\frac{1+1}{e} + \frac{0+1}{1} = -\frac{2}{e} + 1
 \end{aligned}$$

- 23 a** $\int x^2 e^x dx = \int x^2 d e^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - \int e^x \cdot 2x dx = x^2 e^x - \int 2x e^x dx$
b $\int 2x e^x dx = 2x e^x - \int e^x d 2x = 2x e^x - \int e^x \cdot 2 dx = 2x e^x - 2 e^x + c$
c $\int x^2 e^x dx = x^2 e^x - \int 2x e^x dx = x^2 e^x - (2x e^x - 2 e^x + c) = x^2 e^x - 2x e^x + 2 e^x + c = (x^2 - 2x + 2) e^x + c$
 Dus de primitieven van f zijn $F(x) = (x^2 - 2x + 2) e^x + c$

Bladzijde 194

- 24 a** $F(x) = (ax^2 + bx + c) e^x$ geeft
 $F'(x) = (2ax + b) e^x + (ax^2 + bx + c) e^x = (ax^2 + (2a + b)x + b + c) e^x$
 Dit moet gelijk zijn aan $f(x) = (ax^2 + px + q) e^x$.
 Dit geeft $p = 2a + b$ en $q = b + c$.
b $f(x) = (4x^2 - 5x + 6) e^x$, dus $a = 4$, $p = -5$ en $q = 6$.
 $a = 4$ en $2a + b = -5$ geeft $8 + b = -5$, dus $b = -13$.
 $b = -13$ en $b + c = 6$ geeft $-13 + c = 6$, dus $c = 19$.
 Dus de primitieven van $f(x) = (4x^2 - 5x + 6) e^x$ zijn $F(x) = (4x^2 - 13x + 19) e^x + c$.

- 25 a** $\int e^x \sin(x) dx = \int \sin(x) d e^x = e^x \sin(x) - \int e^x d \sin(x) = e^x \sin(x) - \int e^x \cos(x) dx$
 $= e^x \sin(x) - \int \cos(x) d e^x = e^x \sin(x) - (e^x \cos(x) - \int e^x d \cos(x))$
 $= e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x) dx$
 Uit $\int e^x \sin(x) dx = e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x) dx$ volgt $2 \int e^x \sin(x) dx = e^x \sin(x) - e^x \cos(x)$
 dus $\int e^x \sin(x) dx = \frac{1}{2} e^x \sin(x) - \frac{1}{2} e^x \cos(x)$.

De primitieven zijn $G(x) = \frac{1}{2} e^x (\sin(x) - \cos(x)) + c$.

- b** $F(x) = -x^2 \cos(x) + 2x \sin(x) + 2 \cos(x) + c$ geeft
 $F'(x) = -2x \cdot \cos(x) - x^2 \cdot -\sin(x) + 2 \cdot \sin(x) + 2x \cdot \cos(x) - 2 \sin(x) = x^2 \sin(x)$

- 26 a** $F(x) = \int \frac{1}{4} x^2 \cos(x) dx = \int \frac{1}{4} x^2 d \sin(x) = \frac{1}{4} x^2 \sin(x) - \int \sin(x) d \frac{1}{4} x^2 = \frac{1}{4} x^2 \sin(x) - \int \frac{1}{2} x \sin(x) dx$
 $= \frac{1}{4} x^2 \sin(x) + \int \frac{1}{2} x d \cos(x) = \frac{1}{4} x^2 \sin(x) + \frac{1}{2} x \cos(x) - \int \cos(x) d \frac{1}{2} x$
 $= \frac{1}{4} x^2 \sin(x) + \frac{1}{2} x \cos(x) - \int \frac{1}{2} \cos(x) dx = \frac{1}{4} x^2 \sin(x) + \frac{1}{2} x \cos(x) - \frac{1}{2} \sin(x) + c$

- b** $\int e^{-x} \cos(x) dx = \int -\cos(x) d e^{-x} = -\cos(x) e^{-x} + \int e^{-x} d \cos(x) = -e^{-x} \cos(x) - \int e^{-x} \sin(x) dx$
 $= -e^{-x} \cos(x) + \int \sin(x) d e^{-x} = -e^{-x} \cos(x) + e^{-x} \sin(x) - \int e^{-x} d \sin(x)$
 $= -e^{-x} \cos(x) + e^{-x} \sin(x) - \int e^{-x} \cos(x) dx$

Uit $\int e^{-x} \cos(x) dx = -e^{-x} \cos(x) + e^{-x} \sin(x) - \int e^{-x} \cos(x) dx$ volgt $2 \int e^{-x} \cos(x) dx = -e^{-x} \cos(x) + e^{-x} \sin(x)$

dus $\int e^{-x} \cos(x) dx = \frac{1}{2} e^{-x} (\sin(x) - \cos(x))$.

De primitieven zijn $G(x) = \frac{1}{2} e^{-x} (\sin(x) - \cos(x)) + c$.

$$\begin{aligned}
 \text{c} \quad \int e^{2x} \sin(x) dx &= \int e^{2x} d(-\cos(x)) = -e^{2x} \cos(x) + \int \cos(x) d e^{2x} = -e^{2x} \cos(x) + \int \cos(x) \cdot 2 e^{2x} dx \\
 &= -e^{2x} \cos(x) + \int 2 e^{2x} d \sin(x) = -e^{2x} \cos(x) + 2 \int e^{2x} \sin(x) dx - \int \sin(x) d 2 e^{2x} \\
 &= -e^{2x} \cos(x) + 2 \int e^{2x} \sin(x) dx - \int 4 e^{2x} \sin(x) dx \\
 &= -e^{2x} \cos(x) + 2 \int e^{2x} \sin(x) dx - 4 \int e^{2x} \sin(x) dx
 \end{aligned}$$

Uit $\int e^{2x} \sin(x) dx = -e^{2x} \cos(x) + 2 \int e^{2x} \sin(x) dx$ volgt $5 \int e^{2x} \sin(x) dx = -e^{2x} \cos(x) + 2 e^{2x} \sin(x)$

$$\text{dus } \int e^{2x} \sin(x) dx = -\frac{1}{5} e^{2x} \cos(x) + \frac{2}{5} e^{2x} \sin(x).$$

De primitieven zijn $H(x) = \frac{2}{5} e^{2x} \sin(x) - \frac{1}{5} e^{2x} \cos(x) + c = \frac{1}{5} e^{2x} (2 \sin(x) - \cos(x)) + c$.

$$\begin{aligned}
 \text{d} \quad K(x) &= \int \ln^2(x) dx = x \ln^2(x) - \int x d \ln^2(x) = x \ln^2(x) - \int x \cdot 2 \ln(x) \cdot \frac{1}{x} dx = x \ln^2(x) - \int 2 \ln(x) dx \\
 &= x \ln^2(x) - 2(x \ln(x) - x) + c = x \ln^2(x) - 2x \ln(x) + 2x + c
 \end{aligned}$$

Bladzijde 195

$$\begin{aligned}
 \text{27 a} \quad \int (x^2 - x) e^x dx &= \int (x^2 - x) d e^x = (x^2 - x) e^x - \int e^x d(x^2 - x) = (x^2 - x) e^x - \int (2x - 1) e^x dx \\
 &= (x^2 - x) e^x - \int (2x - 1) d e^x = (x^2 - x) e^x - (2x - 1) e^x + \int e^x d(2x - 1) \\
 &= (x^2 - x) e^x - (2x - 1) e^x + \int 2 e^x dx = (x^2 - x) e^x - (2x - 1) e^x + 2 e^x + c \\
 &= (x^2 - 3x + 3) e^x + c
 \end{aligned}$$

$$\int_1^3 (x^2 - x) e^x dx = [(x^2 - 3x + 3) e^x]_1^3 = (9 - 9 + 3) e^3 - (1 - 3 + 3) e^1 = 3e^3 - e$$

$$\begin{aligned}
 \text{b} \quad \int x \ln^2(x) dx &= \int \ln^2(x) d \frac{1}{2} x^2 = \frac{1}{2} x^2 \ln^2(x) - \int \frac{1}{2} x^2 d \ln^2(x) = \frac{1}{2} x^2 \ln^2(x) - \int \frac{1}{2} x^2 \cdot 2 \ln(x) \cdot \frac{1}{x} dx \\
 &= \frac{1}{2} x^2 \ln^2(x) - \int x \ln(x) dx = \frac{1}{2} x^2 \ln^2(x) - \int \ln(x) d \frac{1}{2} x^2 \\
 &= \frac{1}{2} x^2 \ln^2(x) - \frac{1}{2} x^2 \ln(x) + \int \frac{1}{2} x^2 d \ln(x) = \frac{1}{2} x^2 \ln^2(x) - \frac{1}{2} x^2 \ln(x) + \int \frac{1}{2} x^2 \cdot \frac{1}{x} dx \\
 &= \frac{1}{2} x^2 \ln^2(x) - \frac{1}{2} x^2 \ln(x) + \int \frac{1}{2} x dx = \frac{1}{2} x^2 \ln^2(x) - \frac{1}{2} x^2 \ln(x) + \frac{1}{4} x^2 + c \\
 \int_1^e x \ln^2(x) dx &= \left[\frac{1}{2} x^2 \ln^2(x) - \frac{1}{2} x^2 \ln(x) + \frac{1}{4} x^2 \right]_1^e \\
 &= \frac{1}{2} e^2 \ln^2(e) - \frac{1}{2} e^2 \ln(e) + \frac{1}{4} e^2 - \left(\frac{1}{2} \ln^2(1) - \frac{1}{2} \ln(1) + \frac{1}{4} \right) = \frac{1}{4} e^2 - \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{28 a} \quad f(x) &= (2x^2 + x - 1) e^x \text{ geeft } f'(x) = (4x + 1) \cdot e^x + (2x^2 + x - 1) \cdot e^x = (2x^2 + 5x) e^x \\
 f'(x) &= 0 \text{ geeft } (2x^2 + 5x) e^x = 0 \\
 2x^2 + 5x &= 0 \\
 x(2x + 5) &= 0 \\
 x &= 0 \vee x = -2\frac{1}{2}
 \end{aligned}$$

$$\text{max. is } f(-2\frac{1}{2}) = (2 \cdot 6\frac{1}{4} - 2\frac{1}{2} - 1) e^{-2\frac{1}{2}} = \frac{9}{e^2 \sqrt{e}}$$

$$\text{min. is } f(0) = -1$$

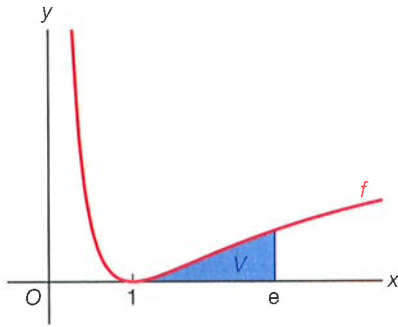
$$\begin{aligned}
 \text{b} \quad f(x) &= 0 \text{ geeft } (2x^2 + x - 1) e^x = 0 \\
 2x^2 + x - 1 &= 0 \\
 D &= 1^2 - 4 \cdot 2 \cdot -1 = 9 \\
 x &= \frac{-1 \pm 3}{4} = \frac{1}{2} \vee x = \frac{-1 - 3}{4} = -1
 \end{aligned}$$

$$\begin{aligned}
 \int (2x^2 + x - 1) e^x dx &= \int (2x^2 + x - 1) d e^x = (2x^2 + x - 1) e^x - \int e^x d(2x^2 + x - 1) \\
 &= (2x^2 + x - 1) e^x - \int (4x + 1) e^x dx = (2x^2 + x - 1) e^x - \int (4x + 1) d e^x \\
 &= (2x^2 + x - 1) e^x - (4x + 1) e^x + \int e^x d(4x + 1) \\
 &= (2x^2 + x - 1) e^x - (4x + 1) e^x + \int 4 e^x dx \\
 &= (2x^2 + x - 1) e^x - (4x + 1) e^x + 4 e^x + c = (2x^2 - 3x + 2) e^x + c
 \end{aligned}$$

$$\int_{-1}^{\frac{1}{2}} (2x^2 + x - 1) e^x dx = [(2x^2 - 3x + 2) e^x]_{-1}^{\frac{1}{2}} = (\frac{1}{2} - 1\frac{1}{2} + 2) e^{\frac{1}{2}} - (2 + 3 + 2) e^{-1} = \sqrt{e} - \frac{7}{e}$$

$$\text{Dus } O(V) = \frac{7}{e} - \sqrt{e}.$$

- 29 a $f(x) = 0$ geeft $\ln^2(x) = 0$
 $\ln(x) = 0$
 $x = 1$



$$\begin{aligned}\int \frac{\ln^2(x)}{\sqrt{x}} dx &= \int 2 \ln^2(x) \cdot \frac{1}{2\sqrt{x}} dx = \int 2 \ln^2(x) d\sqrt{x} = 2\sqrt{x} \cdot \ln^2(x) - \int 2\sqrt{x} d\ln^2(x) \\ &= 2\sqrt{x} \cdot \ln^2(x) - \int 2\sqrt{x} \cdot 2 \ln(x) \cdot \frac{1}{x} dx = 2\sqrt{x} \cdot \ln^2(x) - \int 4 \ln(x) \cdot x^{-\frac{1}{2}} dx \\ &= 2\sqrt{x} \cdot \ln^2(x) - \int 8 \ln(x) dx^{\frac{1}{2}} = 2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + \int 8\sqrt{x} d\ln(x) \\ &= 2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + \int 8x^{\frac{1}{2}} \cdot \frac{1}{x} dx = 2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + \int 8x^{-\frac{1}{2}} dx \\ &= 2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + 16x^{\frac{1}{2}} + c = 2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + 16\sqrt{x} + c\end{aligned}$$

$$\begin{aligned}O(V) &= \int_1^e \frac{\ln^2(x)}{\sqrt{x}} dx = [2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + 16\sqrt{x}]_1^e \\ &= 2\sqrt{e} \cdot \ln^2(e) - 8\sqrt{e} \cdot \ln(e) + 16\sqrt{e} - (2\ln^2(1) - 8\ln(1) + 16) \\ &= 2\sqrt{e} - 8\sqrt{e} + 16\sqrt{e} - 16 = 10\sqrt{e} - 16\end{aligned}$$

b $I(L) = \pi \int_1^e \frac{\ln^4(x)}{x} dx = \pi \int_1^e \ln^4(x) \cdot \frac{1}{x} dx = \pi \int_{x=1}^{x=e} \ln^4(x) d\ln(x) = \pi \int_0^1 u^4 du = \pi \left[\frac{1}{5} u^5 \right]_0^1 = \frac{1}{5} \pi$

K.3 Cyclometrische functies

Bladzijde 197

30 $\int_0^1 \frac{1}{x^2+1} dx = \int_{t=0}^{t=\frac{1}{4}\pi} \frac{1}{\tan^2(t)+1} d\tan(t) = \int_0^{\frac{1}{4}\pi} \frac{1}{\tan^2(t)+1} \cdot (\tan^2(t)+1) dt = \int_0^{\frac{1}{4}\pi} 1 dt = [t]_0^{\frac{1}{4}\pi} = \frac{1}{4}\pi$

Bladzijde 199

- 31 a Het bereik van $f(x) = \arctan(x)$ is $\left(-\frac{1}{2}\pi, \frac{1}{2}\pi\right)$.
 $1\frac{1}{6}\pi$ zit niet in het bereik, dus $\arctan(\frac{1}{3}\sqrt{3}) \neq 1\frac{1}{6}\pi$.
 b $\sqrt{3} > \frac{1}{2}\pi$, dus de vergelijking $\arctan(x) = \sqrt{3}$ heeft geen oplossing.
 c Een primitieve van $f(ax+b)$ is $\frac{1}{a}F(ax+b)$.

32

x	$-\sqrt{3}$	-1	$-\frac{1}{3}\sqrt{3}$	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$
$\arctan(x)$	$-\frac{1}{3}\pi$	$-\frac{1}{4}\pi$	$-\frac{1}{6}\pi$	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$

- 33 a** $\arctan(x) = \frac{1}{3}\pi$
 $x = \tan(\frac{1}{3}\pi)$
 $x = \sqrt{3}$
- b** $\arctan(x-2) = -\frac{1}{4}\pi$
 $x-2 = \tan(-\frac{1}{4}\pi)$
 $x-2 = -1$
 $x = 1$
- c** $\arctan(x^2-1) = \frac{1}{4}\pi$
 $x^2-1 = \tan(\frac{1}{4}\pi)$
 $x^2-1 = 1$
 $x^2 = 2$
 $x = \sqrt{2} \vee x = -\sqrt{2}$
- d** $\arctan(x) = \frac{2}{3}\pi$
geen oplossing, want $\frac{2}{3}\pi > \frac{1}{2}\pi$
- e** $\arctan(x) = \sqrt{2}$
 $x = \tan(\sqrt{2})$
 $x \approx 6,334$
- f** $\arctan(x^2-1) = 1$
 $x^2-1 = \tan(1)$
 $x^2-1 = 1,557...$
 $x^2 = 2,557...$
 $x = \sqrt{2,557...} \vee x = -\sqrt{2,557...}$
 $x \approx 1,599 \vee x = -1,599$

Bladzijde 200

- 34 a** $f(x) = 2 \arctan(\frac{1}{2}x)$ geeft $f'(x) = 2 \cdot \frac{1}{(\frac{1}{2}x)^2 + 1} \cdot \frac{1}{2} = \frac{1}{\frac{1}{4}x^2 + 1} = \frac{4}{x^2 + 4}$
- b** $g(x) = \arctan(x-2)$ geeft $g'(x) = \frac{1}{(x-2)^2 + 1} = \frac{1}{x^2 - 4x + 4 + 1} = \frac{1}{x^2 - 4x + 5}$
- c** $h(x) = \arctan(x^2)$ geeft $h'(x) = \frac{1}{(x^2)^2 + 1} \cdot 2x = \frac{2x}{x^4 + 1}$
- 35 a** $\int_{-\frac{1}{3}\sqrt{3}}^{\sqrt{3}} \frac{1}{x^2 + 1} dx = [\arctan(x)]_{-\frac{1}{3}\sqrt{3}}^{\sqrt{3}} = \arctan(\sqrt{3}) - \arctan(-\frac{1}{3}\sqrt{3}) = \frac{1}{3}\pi - -\frac{1}{6}\pi = \frac{1}{2}\pi$
- b** $\int_{-2}^{-1} \frac{1}{(x+1)^2 + 1} dx = [\arctan(x+1)]_{-2}^{-1} = \arctan(0) - \arctan(-1) = 0 - -\frac{1}{4}\pi = \frac{1}{4}\pi$
- c** $\int_0^{\frac{1}{3}\sqrt{3}} \frac{3}{9x^2 + 1} dx = \int_0^{\frac{1}{3}\sqrt{3}} \frac{3}{(3x)^2 + 1} dx = [3 \cdot \frac{1}{3} \arctan(3x)]_0^{\frac{1}{3}\sqrt{3}} = [\arctan(3x)]_0^{\frac{1}{3}\sqrt{3}}$
 $= \arctan(\sqrt{3}) - \arctan(0) = \frac{1}{3}\pi - 0 = \frac{1}{3}\pi$
- 36 a** $\arctan(2x-1) = \frac{1}{6}\pi$
 $2x-1 = \tan(\frac{1}{6}\pi)$
 $2x-1 = \frac{1}{3}\sqrt{3}$
 $2x = 1 + \frac{1}{3}\sqrt{3}$
 $x = \frac{1}{2} + \frac{1}{6}\sqrt{3}$
- b** $\arctan(\frac{1}{2}\pi + \sqrt{x}) = \sqrt{2}$
 $\frac{1}{2}\pi + \sqrt{x} = \tan(\sqrt{2})$
 $\sqrt{x} = \tan(\sqrt{2}) - \frac{1}{2}\pi$
 $x = (\tan(\sqrt{2}) - \frac{1}{2}\pi)^2 \approx 2,63$
- c** $f(x) = \arctan(2x^2+1)$ geeft $f'(x) = \frac{1}{(2x^2+1)^2 + 1} \cdot 4x = \frac{4x}{4x^4 + 4x^2 + 1 + 1} = \frac{2x}{2x^4 + 2x^2 + 1}$
- d** $\int_{\frac{1}{5}}^{\frac{1}{3}\sqrt{3}} \frac{10}{25x^2 + 1} dx = \int_{\frac{1}{5}}^{\frac{1}{3}\sqrt{3}} \frac{10}{(5x)^2 + 1} dx = [10 \cdot \frac{1}{5} \arctan(5x)]_{\frac{1}{5}}^{\frac{1}{3}\sqrt{3}} = 2 \arctan(\sqrt{3}) - 2 \arctan(1) = 2 \cdot \frac{1}{3}\pi - 2 \cdot \frac{1}{4}\pi = \frac{1}{6}\pi$
- 37 a** $f(x) = \frac{10}{x^2 + 4} = \frac{2\frac{1}{2}}{\frac{1}{4}x^2 + 1} = \frac{2\frac{1}{2}}{(\frac{1}{2}x)^2 + 1}$
Dus $a = 2\frac{1}{2}$ en $b = \frac{1}{2}$.
- b** $F(x) = 2\frac{1}{2} \cdot \frac{1}{\frac{1}{2}} \cdot \arctan(\frac{1}{2}x) + c = 5 \arctan(\frac{1}{2}x) + c$

Bladzijde 201

$$38 \quad a \quad F(x) = \int \frac{5}{32x^2 + 2} dx = \int \frac{2^{\frac{1}{2}}}{16x^2 + 1} dx = \int \frac{2^{\frac{1}{2}}}{(4x)^2 + 1} dx = 2^{\frac{1}{2}} \cdot \frac{1}{4} \arctan(4x) + c = \frac{5}{8} \arctan(4x) + c$$

$$b \quad G(x) = \int \frac{4}{x^2 + 4} dx = \int \frac{1}{\frac{1}{4}x^2 + 1} dx = \int \frac{1}{(\frac{1}{2}x)^2 + 1} dx = 2 \arctan(\frac{1}{2}x) + c$$

$$c \quad H(x) = \int \frac{5}{x^2 + 6x + 10} dx = \int \frac{5}{(x+3)^2 - 9 + 10} dx = \int \frac{5}{(x+3)^2 + 1} dx = 5 \arctan(x+3) + c$$

$$d \quad J(x) = \int \frac{3}{x^2 + 4x + 5} dx = \int \frac{3}{(x+2)^2 - 4 + 5} dx = \int \frac{3}{(x+2)^2 + 1} dx = 3 \arctan(x+2) + c$$

$$39 \quad a \quad \int_0^1 \frac{3}{x^2 + 3} dx = \int_0^1 \frac{1}{\frac{1}{3}x^2 + 1} dx = \int_0^1 \frac{1}{\left(\frac{x}{\sqrt{3}}\right)^2 + 1} dx = \left[\sqrt{3} \cdot \arctan\left(\frac{x}{\sqrt{3}}\right) \right]_0^1$$

$$= \sqrt{3} \cdot \arctan\left(\frac{1}{\sqrt{3}}\right) - \sqrt{3} \cdot \arctan\left(\frac{0}{\sqrt{3}}\right) = \sqrt{3} \cdot \arctan\left(\frac{1}{\sqrt{3}}\right) - \sqrt{3} \cdot \arctan(0)$$

$$= \sqrt{3} \cdot \frac{1}{6}\pi - 0 = \frac{1}{6}\pi\sqrt{3}$$

$$b \quad \int_0^1 \frac{2x}{x^4 + 1} dx = \int_{x=0}^{x=1} \frac{1}{(x^2)^2 + 1} dx^2 = [\arctan(x^2)]_0^1 = \arctan(1) - \arctan(0) = \frac{1}{4}\pi$$

$$c \quad \int_3^6 \frac{5}{x^2 - 6x + 18} dx = \int_3^6 \frac{5}{(x-3)^2 - 9 + 18} dx = \int_3^6 \frac{5}{(x-3)^2 + 9} dx = \int_3^6 \frac{\frac{5}{9}}{\frac{1}{9}(x-3)^2 + 1} dx = \int_3^6 \frac{\frac{5}{9}}{(\frac{1}{3}(x-3))^2 + 1} dx$$

$$= \left[\frac{5}{9} \cdot 3 \arctan\left(\frac{1}{3}(x-3)\right) \right]_3^6 = \frac{5}{3} \arctan(1) - \frac{5}{3} \arctan(0) = \frac{5}{3} \cdot \frac{1}{4}\pi = \frac{5}{12}\pi$$

$$d \quad \int_0^{\sqrt{3}} \frac{\arctan(x)}{x^2 + 1} dx = \int_0^{\sqrt{3}} \arctan(x) \cdot \frac{1}{x^2 + 1} dx = \int_{x=0}^{x=\sqrt{3}} \arctan(x) d \arctan(x) = \int_0^{\frac{1}{2}\pi} u du = \left[\frac{1}{2} u^2 \right]_0^{\frac{1}{2}\pi}$$

$$= \frac{1}{2} \cdot \frac{1}{9}\pi^2 - 0 = \frac{1}{18}\pi^2$$

Bladzijde 202

$$40 \quad a \quad \int_0^{\frac{1}{2}} \frac{2}{4x^2 + 9} dx = \int_0^{\frac{1}{2}} \frac{\frac{2}{9}}{\frac{4}{9}x^2 + 1} dx = \int_0^{\frac{1}{2}} \frac{\frac{2}{9}}{(\frac{2}{3}x)^2 + 1} dx = \left[\frac{2}{9} \cdot \frac{3}{2} \arctan\left(\frac{2}{3}x\right) \right]_0^{\frac{1}{2}} = \frac{1}{3} \arctan(1) - \frac{1}{3} \arctan(0)$$

$$= \frac{1}{3} \cdot \frac{1}{4}\pi = \frac{1}{12}\pi$$

$$b \quad \int_0^{\frac{1}{2}\ln(3)} \frac{e^x}{e^{2x} + 1} dx = \int_{x=0}^{x=\frac{1}{2}\ln(3)} \frac{1}{(e^x)^2 + 1} d e^x = \int_1^{\sqrt{3}} \frac{1}{u^2 + 1} du = [\arctan(u)]_1^{\sqrt{3}} = \arctan(\sqrt{3}) - \arctan(1) = \frac{1}{3}\pi - \frac{1}{4}\pi = \frac{1}{12}\pi$$

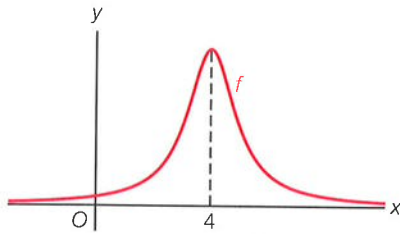
$$c \quad \int_0^1 \frac{1}{4x^2 - 4x + 2} dx = \int_0^1 \frac{1}{(2x-1)^2 - 1 + 2} dx = \int_0^1 \frac{1}{(2x-1)^2 + 1} dx = \left[\frac{1}{2} \arctan(2x-1) \right]_0^1$$

$$= \frac{1}{2} \arctan(1) - \frac{1}{2} \arctan(-1) = \frac{1}{2} \cdot \frac{1}{4}\pi - \frac{1}{2} \cdot -\frac{1}{4}\pi = \frac{1}{4}\pi$$

$$d \quad \int_0^{\frac{1}{2}\pi} \frac{\sin(x)}{\cos^2(x) + 1} dx = \int_{x=0}^{x=\frac{1}{2}\pi} -\frac{1}{\cos^2(x) + 1} d \cos(x) = \int_1^0 -\frac{1}{u^2 + 1} du = [-\arctan(u)]_1^0$$

$$= -\arctan(0) + \arctan(1) = \frac{1}{4}\pi$$

41 a $f(x) = \frac{10}{x^2 - 8x + 17}$ geeft $f'(x) = \frac{(x^2 - 8x + 17) \cdot 0 - 10(2x - 8)}{(x^2 - 8x + 17)^2} = \frac{-20x + 80}{(x^2 - 8x + 17)^2}$
 $f'(x) = 0$ geeft $-20x + 80 = 0$
 $-20x = -80$
 $x = 4$



max. is $f(4) = \frac{10}{16 - 32 + 17} = 10$

b $O(V) = \int_4^5 \frac{10}{x^2 - 8x + 17} dx = \int_4^5 \frac{10}{(x - 4)^2 - 16 + 17} dx = \int_4^5 \frac{10}{(x - 4)^2 + 1} dx = [10 \arctan(x - 4)]_4^5$
 $= 10 \arctan(1) - 10 \arctan(0) = 10 \cdot \frac{1}{4} \pi = 2\frac{1}{2} \pi$

c $O(W_p) = 10$ geeft $\int_4^p f(x) dx = 10$
 $[10 \arctan(x - 4)]_4^p = 10$
 $10 \arctan(p - 4) - 10 \arctan(0) = 10$
 $\arctan(p - 4) = 1$
 $p - 4 = \tan(1)$
 $p = 4 + \tan(1)$

42 $\int \frac{4}{x \ln^2(x) + x} dx = \int \frac{1}{x} \cdot \frac{4}{\ln^2(x) + 1} dx = \int \frac{4}{\ln^2(x) + 1} d \ln(x) = \int \frac{4}{u^2 + 1} du = 4 \arctan(u) + c = 4 \arctan(\ln(x)) + c$

$O(V_p) = \int_1^p f(x) dx = [4 \arctan(\ln(x))]_1^p = 4 \arctan(\ln(p)) - 4 \arctan(0) = 4 \arctan(\ln(p))$

$O(V_p) = \pi$ geeft $4 \arctan(\ln(p)) = \pi$

$\arctan(\ln(p)) = \frac{1}{4} \pi$

$\ln(p) = \tan(\frac{1}{4} \pi)$

$\ln(p) = 1$

$p = e$

43 $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1 - x^2}} dx = \int_{t=0}^{t=\frac{1}{6}\pi} \frac{1}{\sqrt{1 - \sin^2(t)}} d \sin(t) = \int_0^{\frac{1}{6}\pi} \frac{1}{\sqrt{\cos^2(t)}} \cdot \cos(t) dt = \int_0^{\frac{1}{6}\pi} \frac{1}{\cos(t)} \cdot \cos(t) dt = \int_0^{\frac{1}{6}\pi} 1 dt = [t]_0^{\frac{1}{6}\pi} = \frac{1}{6} \pi$

Bladzijde 204

44

x	-1	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
$\arcsin(x)$	$-\frac{1}{2}\pi$	$-\frac{1}{3}\pi$	$-\frac{1}{4}\pi$	$-\frac{1}{6}\pi$	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$

45 a $\arcsin(x) = \frac{1}{2} \pi$

$x = \sin(\frac{1}{2} \pi)$

$x = 1$

b $\arcsin(x) = -\frac{1}{6} \pi$

$x = \sin(-\frac{1}{6} \pi)$

$x = -\frac{1}{2}$

- c $\arcsin(x) = 2$
geen oplossing, want $2 > \frac{1}{2}\pi$
- d $3 \arcsin(x - \sqrt{3}) = \pi$
 $\arcsin(x - \sqrt{3}) = \frac{1}{3}\pi$
 $x - \sqrt{3} = \sin(\frac{1}{3}\pi)$
 $x - \sqrt{3} = \frac{1}{2}\sqrt{3}$
 $x = 1\frac{1}{2}\sqrt{3} \approx 2,598$

46 a $\int_{\frac{1}{6}}^{\frac{1}{6}\sqrt{3}} \frac{1}{\sqrt{1-9x^2}} dx = \int_{\frac{1}{6}}^{\frac{1}{6}\sqrt{3}} \frac{1}{\sqrt{1-(3x)^2}} dx = \left[\frac{1}{3} \arcsin(3x) \right]_{\frac{1}{6}}^{\frac{1}{6}\sqrt{3}} = \frac{1}{3} \arcsin(\frac{1}{2}\sqrt{3}) - \frac{1}{3} \arcsin(\frac{1}{2})$
 $= \frac{1}{3} \cdot \frac{1}{3}\pi - \frac{1}{3} \cdot \frac{1}{6}\pi = \frac{1}{9}\pi - \frac{1}{18}\pi = \frac{1}{18}\pi$

b $\int_{-1\frac{1}{2}\sqrt{2}}^{1\frac{1}{2}\sqrt{2}} \frac{1}{\sqrt{9-x^2}} dx = \int_{-1\frac{1}{2}\sqrt{2}}^{1\frac{1}{2}\sqrt{2}} \frac{\frac{1}{3}}{\sqrt{1-\frac{1}{9}x^2}} dx = \int_{-1\frac{1}{2}\sqrt{2}}^{1\frac{1}{2}\sqrt{2}} \frac{\frac{1}{3}}{\sqrt{1-(\frac{1}{3}x)^2}} dx = \left[\arcsin(\frac{1}{3}x) \right]_{-1\frac{1}{2}\sqrt{2}}^{1\frac{1}{2}\sqrt{2}}$
 $= \arcsin(\frac{1}{3} \cdot 1\frac{1}{2}\sqrt{2}) - \arcsin(\frac{1}{3} \cdot -1\frac{1}{2}\sqrt{2}) = \arcsin(\frac{1}{2}\sqrt{2}) - \arcsin(-\frac{1}{2}\sqrt{2})$
 $= \frac{1}{4}\pi - (-\frac{1}{4}\pi) = \frac{1}{2}\pi$

c $\int_0^{\frac{1}{2}\sqrt{2}} \frac{x}{\sqrt{1-x^4}} dx = \int_{x=0}^{x=\frac{1}{2}\sqrt{2}} \frac{1}{2} \cdot \frac{1}{\sqrt{1-(x^2)^2}} dx^2 = \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-u^2}} du = \left[\frac{1}{2} \arcsin(u) \right]_0^{\frac{1}{2}}$
 $= \frac{1}{2} \arcsin(\frac{1}{2}) - \frac{1}{2} \arcsin(0) = \frac{1}{2} \cdot \frac{1}{6}\pi - 0 = \frac{1}{12}\pi$

d $\int_0^1 \frac{x}{\sqrt{4-x^4}} dx = \int_0^1 \frac{\frac{1}{2}x}{\sqrt{1-\frac{1}{4}x^4}} dx = \int_{x=0}^{x=1} \frac{\frac{1}{2}}{\sqrt{1-(\frac{1}{2}x^2)^2}} d\frac{1}{2}x^2 = \int_0^{\frac{1}{2}} \frac{\frac{1}{2}}{\sqrt{1-u^2}} du = \left[\frac{1}{2} \arcsin(u) \right]_0^{\frac{1}{2}}$
 $= \frac{1}{2} \arcsin(\frac{1}{2}) - \frac{1}{2} \arcsin(0) = \frac{1}{2} \cdot \frac{1}{6}\pi - 0 = \frac{1}{12}\pi$

47 a $F(x) = \int \arctan(x) dx = x \arctan(x) - \int x d \arctan(x) = x \arctan(x) - \int x \cdot \frac{1}{x^2+1} dx$
 $= x \arctan(x) - \int \frac{1}{2} \cdot \frac{1}{x^2+1} d(x^2+1) = x \arctan(x) - \frac{1}{2} \ln(x^2+1) + c$

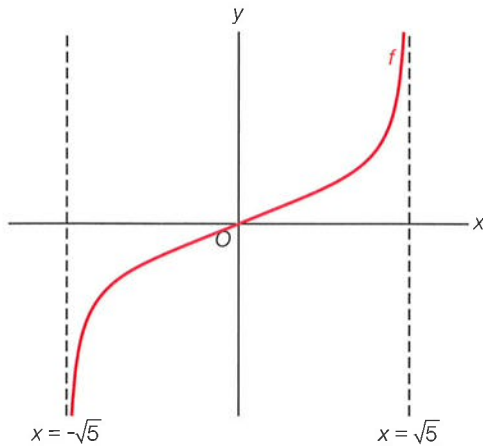
b $G(x) = \int \arcsin(x) dx = x \arcsin(x) - \int x d \arcsin(x) = x \arcsin(x) - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$
 $= x \arcsin(x) - \int -\frac{1}{2\sqrt{1-x^2}} d(1-x^2) = x \arcsin(x) + \sqrt{1-x^2} + c$

Bladzijde 205

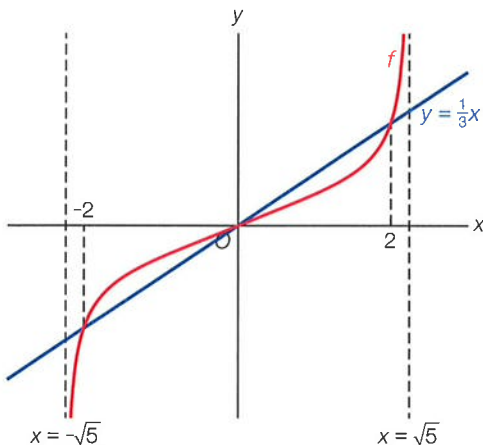
48 a $F(x) = \int \frac{\arcsin(x)}{\sqrt{1-x^2}} dx = \int \arcsin(x) \cdot \frac{1}{\sqrt{1-x^2}} dx = \int \arcsin(x) d \arcsin(x) = \frac{1}{2} \arcsin^2(x) + c$

b $G(x) = \int \frac{1}{x\sqrt{1-\ln^2(x)}} dx = \int \frac{1}{\sqrt{1-\ln^2(x)}} \cdot \frac{1}{x} dx = \int \frac{1}{\sqrt{1-\ln^2(x)}} d \ln(x) = \arcsin(\ln(x)) + c$

49 a $25 - x^4 > 0$
 $x^4 < 25$
 $-5 < x^2 < 5$
 $x^2 < 5$
 $-\sqrt{5} < x < \sqrt{5}$
Dus $D_f = (-\sqrt{5}, \sqrt{5})$.



b $f(x) = \frac{1}{3}x$ geeft $\frac{x}{\sqrt{25-x^4}} = \frac{x}{3}$
 $x = 0 \vee \sqrt{25-x^4} = 3$
 $x = 0 \vee 25 - x^4 = 9$
 $x = 0 \vee x^4 = 16$
 $x = 0 \vee x = 2 \vee x = -2$



$f(x) \leq \frac{1}{3}x$ geeft $-\sqrt{5} < x \leq -2 \vee 0 \leq x \leq 2$

c $O(V) = \int_0^{\frac{1}{2}\sqrt{10}} \frac{x}{\sqrt{25-x^4}} dx = \int_0^{\frac{1}{2}\sqrt{10}} \frac{\frac{1}{5}x}{\sqrt{1-\frac{1}{25}x^4}} dx = \int_0^{\frac{1}{2}\sqrt{10}} \frac{\frac{1}{5}x}{\sqrt{1-(\frac{1}{5}x^2)^2}} dx = \int_{x=0}^{x=\frac{1}{2}\sqrt{10}} \frac{\frac{1}{2}}{\sqrt{1-(\frac{1}{5}x^2)^2}} d\frac{1}{5}x^2$
 $= \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-u^2}} du = \left[\frac{1}{2} \arcsin(u) \right]_0^{\frac{1}{2}} = \frac{1}{2} \arcsin\left(\frac{1}{2}\right) - \frac{1}{2} \arcsin(0) = \frac{1}{2} \cdot \frac{1}{6} \pi = \frac{1}{12} \pi$

K.4 Breuksplitsen

Bladzijde 207

50 a $F(x) = \int \frac{2x}{x^2+1} dx = \int \frac{1}{x^2+1} d(x^2+1) = \ln(x^2+1)$

$G(x) = \int \frac{1}{x^2+1} dx = \arctan(x)$

b Uitdelen geeft $h(x) = \frac{2x+1}{x^2+1} = \frac{2x}{x^2+1} + \frac{1}{x^2+1} = f(x) + g(x)$.

Dus $H(x) = \ln(x^2+1) + \arctan(x) + c$.

51 a Uitdelen geeft $f(x) = \frac{2x+5}{x+1} = \frac{2x}{x+1} + \frac{5}{x+1}$.

f is zo niet te primitiveren, want er is geen primitieve te geven van $\frac{2x}{x+1}$.

b $f(x) = 2 + \frac{3}{x+1} = \frac{2x+2}{x+1} + \frac{3}{x+1} = \frac{2x+2+3}{x+1} = \frac{2x+5}{x+1}$

Dus de splitsing is correct.

$$F(x) = \int \left(2 + \frac{3}{x+1} \right) dx = 2x + 3 \ln|x+1| + c$$

Bladzijde 209

52 a $x+1 / 2x+1 \setminus 2$

$$\frac{2x+2}{-1}$$

Dus $\frac{2x+1}{x+1} = 2 - \frac{1}{x+1}$.

$$F(x) = \int \frac{2x+1}{x+1} dx = \int \left(2 - \frac{1}{x+1} \right) dx = 2x - \ln|x+1| + c$$

b $x+1 / x \setminus 1$

$$\frac{x+1}{-1}$$

Dus $\frac{x}{x+1} = 1 - \frac{1}{x+1}$.

$$F(x) = \int \frac{x}{x+1} dx = \int \left(1 - \frac{1}{x+1} \right) dx = x - \ln|x+1| + c$$

c $2x+1 / x+1 \setminus \frac{1}{2}$

$$\frac{x+\frac{1}{2}}{\frac{1}{2}}$$

Dus $\frac{x+1}{2x+1} = \frac{1}{2} + \frac{\frac{1}{2}}{2x+1}$.

$$F(x) = \int \frac{x+1}{2x+1} dx = \int \left(\frac{1}{2} + \frac{\frac{1}{2}}{2x+1} \right) dx = \frac{1}{2}x + \frac{1}{4} \ln|2x+1| + c$$

d $f(x) = \frac{2-x}{x+1} = \frac{-x+2}{x+1}$
 $x+1 / -x+2 \setminus -1$

$$\frac{-x-1}{3}$$

Dus $\frac{2-x}{x+1} = -1 + \frac{3}{x+1}$.

$$F(x) = \int \frac{2-x}{x+1} dx = \int \left(-1 + \frac{3}{x+1} \right) dx = -x + 3 \ln|x+1| + c$$

53 a $x-1 / x^2-2x+3 \setminus x-1$

$$\frac{x^2-x}{-x+3}$$

$$\frac{-x+1}{2}$$

Dus $\frac{x^2-2x+3}{x-1} = x-1 + \frac{2}{x-1}$.

$$\begin{aligned} \int_2^3 \frac{x^2-2x+3}{x-1} dx &= \int_2^3 \left(x-1 + \frac{2}{x-1} \right) dx = \left[\frac{1}{2}x^2 - x + 2 \ln|x-1| \right]_2^3 = 4\frac{1}{2} - 3 + 2 \ln(2) - (2 - 2 + 2 \ln(1)) \\ &= 1\frac{1}{2} + 2 \ln(2) \end{aligned}$$

$$\begin{array}{r} x+1 \text{ / } -2x^2-x \quad \backslash -2x+1 \\ \underline{-2x^2-2x} \\ x \\ \underline{x+1} \\ -1 \end{array}$$

$$\text{Dus } \frac{-2x^2-x}{x+1} = -2x+1 - \frac{1}{x+1}.$$

$$\begin{aligned} \int_{-4}^{-2} \frac{-2x^2-x}{x+1} dx &= \int_{-4}^{-2} \left(-2x+1 - \frac{1}{x+1} \right) dx = \left[-x^2+x - \ln|x+1| \right]_{-4}^{-2} \\ &= -4-2-\ln(1) - (-16-4-\ln(3)) = 14+\ln(3) \end{aligned}$$

$$\begin{array}{r} 54 \quad a \quad f(x) = \frac{3-4x}{2x+1} = \frac{-4x+3}{2x+1} \\ 2x+1 \text{ / } -4x+3 \quad \backslash -2 \\ \underline{-4x-2} \\ 5 \end{array}$$

$$\text{Dus } \frac{3-4x}{2x+1} = -2 + \frac{5}{2x+1}.$$

$$F(x) = \int \frac{3-4x}{2x+1} dx = \int \left(-2 + \frac{5}{2x+1} \right) dx = -2x + \frac{5}{2} \ln|2x+1| + c$$

$$\begin{array}{r} g(x) = \frac{6x-1}{1-2x} = \frac{6x-1}{-2x+1} \\ -2x+1 \text{ / } 6x-1 \quad \backslash -3 \\ \underline{6x-3} \\ 2 \end{array}$$

$$\text{Dus } \frac{6x-1}{1-2x} = -3 + \frac{2}{1-2x}.$$

$$G(x) = \int \frac{6x-1}{1-2x} dx = \int \left(-3 + \frac{2}{1-2x} \right) dx = -3x - \ln|1-2x| + c$$

$$\begin{array}{r} b \quad 2x-1 \text{ / } 2x^2-5x \quad \backslash x-2 \\ \underline{2x^2-x} \\ -4x \\ \underline{-4x+2} \\ -2 \end{array}$$

$$\text{Dus } \frac{2x^2-5x}{2x-1} = x-2 - \frac{2}{2x-1}.$$

$$\begin{aligned} \int_3^6 \frac{2x^2-5x}{2x-1} dx &= \int_3^6 \left(x-2 - \frac{2}{2x-1} \right) dx = \left[\frac{1}{2}x^2 - 2x - \ln|2x-1| \right]_3^6 = 18-12-\ln(11) - \left(\frac{1}{2} - 6 - \ln(5) \right) \\ &= 6 - \ln(11) + 1\frac{1}{2} + \ln(5) = 7\frac{1}{2} + \ln\left(\frac{5}{11}\right) \end{aligned}$$

$$\begin{array}{l} 55 \quad a \quad f(x) = \frac{x^3+x}{x+1} \text{ geeft} \\ f'(x) = \frac{(x+1) \cdot (3x^2+1) - (x^3+x) \cdot 1}{(x+1)^2} = \frac{3x^3+x+3x^2+1-x^3-x}{(x+1)^2} = \frac{2x^3+3x^2+1}{(x+1)^2} \end{array}$$

$$f'(0) = 1, \text{ dus } k: y = x.$$

$$f(x) = x \text{ geeft } \frac{x^3+x}{x+1} = x$$

$$x \cdot \frac{x^2+1}{x+1} = x$$

$$x = 0 \vee \frac{x^2+1}{x+1} = 1$$

$$x = 0 \vee x^2+1 = x+1$$

$$x = 0 \vee x^2 = x$$

$$x = 0 \vee x = 0 \vee x = 1$$

$$f(1) = 1, \text{ dus } A(1, 1).$$

$$\begin{array}{r} b \quad x + 1 / x^3 + x \quad \setminus x^2 - x + 2 \\ \quad \frac{x^3 + x^2}{-x^2 + x} \\ \quad \quad \frac{-x^2 - x}{2x} \\ \quad \quad \quad \frac{2x + 2}{-2} \end{array}$$

$$\text{Dus } \frac{x^3 + x}{x + 1} = x^2 - x + 2 - \frac{2}{x + 1}.$$

$$\begin{aligned} O(V) &= \int_0^2 \frac{x^3 + x}{x + 1} dx = \int_0^2 \left(x^2 - x + 2 - \frac{2}{x + 1} \right) dx = \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x - 2 \ln|x + 1| \right]_0^2 \\ &= \frac{8}{3} - 2 + 4 - 2 \ln(3) - (0 - 0 + 0 - 2 \ln(1)) = 4\frac{2}{3} - 2 \ln(3) \end{aligned}$$

$$56 \quad a \quad f(x) = \frac{3x + 1}{x + 1} = \frac{3(x + 1) - 3 + 1}{x + 1} = 3 - \frac{2}{x + 1}$$

$$\text{Dus } F(x) = 3x - 2 \ln|x + 1| + c.$$

$$b \quad g(x) = \frac{4x}{2x - 1} = \frac{2(2x - 1) + 2}{2x - 1} = 2 + \frac{2}{2x - 1}$$

$$\text{Dus } G(x) = 2x + \ln|2x - 1| + c.$$

Bladzijde 210

$$57 \quad a \quad f(x) \text{ is voor elke } x \text{ gedefinieerd, want de noemer } x^2 + 4x + 5 \text{ heeft geen nulpunten.}$$

$x^2 + 4x + 4 = (x + 2)^2$ heeft één nulpunt, namelijk -2 , dus de grafiek van g heeft één verticale asymptoot.

$x^2 + 4x + 3 = (x + 1)(x + 3)$ heeft twee nulpunten, namelijk -1 en -3 , dus de grafiek van h heeft twee verticale asymptoten.

$$b \quad x^2 + 4x + 5 = (x + 2)^2 - 4 + 5 = (x + 2)^2 + 1$$

$$c \quad \int \frac{2x + 4}{x^2 + 4x + 5} dx = \int \frac{1}{x^2 + 4x + 5} d(x^2 + 4x + 5) = \ln(x^2 + 4x + 5) + c$$

$$\int \frac{c}{(x + 2)^2 + 1} dx = c \arctan(x + 2) + d$$

$$d \quad \int \frac{a}{x + 2} dx = a \ln|x + 2| + c$$

$$\int \frac{b}{(x + 2)^2} dx = \int b(x + 2)^{-2} dx = -b(x + 2)^{-1} + c = \frac{-b}{x + 2} + c$$

$$e \quad \int \frac{a}{x + 1} dx = a \ln|x + 1| + c$$

$$\int \frac{b}{x + 3} dx = b \ln|x + 3| + c$$

Bladzijde 213

$$58 \quad p(x) = x^2 - 3x + 5, q(x) = 1, l(x) = x + 1, b = -4 \text{ en } c = 4.$$

$$59 \quad a \quad x^2 + 1 / x^2 + x \quad \setminus 1$$

$$\frac{x^2 + 1}{x - 1}$$

$$f(x) = \frac{x^2 + x}{x^2 + 1} = 1 + \frac{x - 1}{x^2 + 1} = 1 + \frac{\frac{1}{2} \cdot 2x - 1}{x^2 + 1} = 1 + \frac{\frac{1}{2} \cdot 2x}{x^2 + 1} - \frac{1}{x^2 + 1}$$

$$\int \frac{\frac{1}{2} \cdot 2x}{x^2 + 1} dx = \int \frac{\frac{1}{2}}{x^2 + 1} d(x^2 + 1) = \frac{1}{2} \ln(x^2 + 1) + c$$

$$\text{Dus } F(x) = x + \frac{1}{2} \ln(x^2 + 1) - \arctan(x) + c.$$

$$\text{b } x^2 - 6x + 9 / x^2 - 6x + 8 \setminus 1$$

$$\frac{x^2 - 6x + 9}{-1} -$$

$$f(x) = \frac{x^2 - 6x + 8}{x^2 - 6x + 9} = 1 - \frac{1}{x^2 - 6x + 9} = 1 - \frac{1}{(x-3)^2} = 1 - (x-3)^{-2} \text{ geeft}$$

$$F(x) = x + (x-3)^{-1} + c = x + \frac{1}{x-3} + c$$

$$\text{c } x^2 + 2x - 3 / x^3 \setminus x - 2$$

$$\frac{x^3 + 2x^2 - 3x}{-2x^2 + 3x} -$$

$$\frac{-2x^2 - 4x + 6}{7x - 6} -$$

$$f(x) = \frac{x^3}{x^2 + 2x - 3} = x - 2 + \frac{7x - 6}{x^2 + 2x - 3} = x - 2 + \frac{7x - 6}{(x-1)(x+3)} = x - 2 + \frac{a}{x-1} + \frac{b}{x+3}$$

$$= x - 2 + \frac{a(x+3) + b(x-1)}{(x-1)(x+3)} = x - 2 + \frac{ax + 3a + bx - b}{(x-1)(x+3)} = x - 2 + \frac{(a+b)x + 3a - b}{(x-1)(x+3)}$$

$$\begin{cases} a + b = 7 \\ 3a - b = -6 \\ 4a = 1 \end{cases} \Rightarrow \begin{cases} \frac{1}{4} + b = 7 \\ b = 6\frac{3}{4} \end{cases}$$

$$\text{Dus } f(x) = x - 2 + \frac{\frac{1}{4}}{x-1} + \frac{6\frac{3}{4}}{x+3}.$$

$$\text{Dit geeft } F(x) = \frac{1}{2}x^2 - 2x + \frac{1}{4}\ln|x-1| + 6\frac{3}{4}\ln|x+3| + c.$$

$$\text{60 a } f(x) = \frac{x^3 + x}{x^2 + 1} = \frac{x(x^2 + 1)}{x^2 + 1} = x \text{ geeft } F(x) = \frac{1}{2}x^2 + c$$

$$\text{b } x^2 - 6x + 9 / x^3 + x \setminus x + 6$$

$$\frac{x^3 - 6x^2 + 9x}{6x^2 - 8x} -$$

$$\frac{6x^2 - 36x + 54}{28x - 54} -$$

$$f(x) = \frac{x^3 + x}{x^2 - 6x + 9} = x + 6 + \frac{28x - 54}{x^2 - 6x + 9} = x + 6 + \frac{28(x-3) + 84 - 54}{(x-3)^2} = x + 6 + \frac{28}{x-3} + \frac{30}{(x-3)^2}$$

$$= x + 6 + \frac{28}{x-3} + 30(x-3)^{-2} \text{ geeft}$$

$$F(x) = \frac{1}{2}x^2 + 6x + 28\ln|x-3| - 30(x-3)^{-1} + c = \frac{1}{2}x^2 + 6x + 28\ln|x-3| - \frac{30}{x-3} + c$$

$$\text{c } 2x^2 + x / 2x^3 + 1 \setminus x - \frac{1}{2}$$

$$\frac{2x^3 + x^2}{-x^2} -$$

$$\frac{-x^2 - \frac{1}{2}x}{\frac{1}{2}x + 1} -$$

$$f(x) = \frac{2x^3 + 1}{2x^2 + x} = x - \frac{1}{2} + \frac{\frac{1}{2}x + 1}{2x^2 + x} = x - \frac{1}{2} + \frac{\frac{1}{2}x + 1}{x(2x + 1)} = x - \frac{1}{2} + \frac{a}{x} + \frac{b}{2x + 1}$$

$$= x - \frac{1}{2} + \frac{a(2x + 1) + bx}{x(2x + 1)} = x - \frac{1}{2} + \frac{2ax + a + bx}{x(2x + 1)} = x - \frac{1}{2} + \frac{(2a + b)x + a}{x(2x + 1)}$$

$$\begin{cases} 2a + b = \frac{1}{2} \\ a = 1 \end{cases} \Rightarrow \begin{cases} 2 + b = \frac{1}{2} \\ b = -1\frac{1}{2} \end{cases}$$

$$\text{Dus } f(x) = x - \frac{1}{2} + \frac{1}{x} - \frac{1\frac{1}{2}}{2x + 1}.$$

$$\text{Dit geeft } F(x) = \frac{1}{2}x^2 - \frac{1}{2}x + \ln|x| - \frac{3}{4}\ln|2x + 1| + c.$$

61 a $x^2 + 4 / x^3 \quad \setminus x$

$$\frac{x^3 + 4x}{-4x}$$

 Dus $\frac{x^3}{x^2 + 4} = x - \frac{4x}{x^2 + 4}$

$$\int \frac{4x}{x^2 + 4} dx = \int \frac{2}{x^2 + 4} d(x^2 + 4) = 2 \ln(x^2 + 4) + c$$

 Dus $\int_0^2 \frac{x^3}{x^2 + 4} dx = \int_0^2 \left(x - \frac{4x}{x^2 + 4} \right) dx = \left[\frac{1}{2}x^2 - 2 \ln(x^2 + 4) \right]_0^2 = 2 - 2 \ln(8) - (0 - 2 \ln(4))$

$$= 2 - 2 \ln(8) + 2 \ln(4) = 2 - 2(\ln(8) - \ln(4)) = 2 - 2 \ln(2)$$

b $x^2 + 4x + 4 / x^3 \quad \setminus x - 4$

$$\frac{x^3 + 4x^2 + 4x}{-4x^2 - 4x}$$

$$\frac{-4x^2 - 16x - 16}{12x + 16}$$

 Dus $\frac{x^3}{x^2 + 4x + 4} = x - 4 + \frac{12x + 16}{x^2 + 4x + 4} = x - 4 + \frac{12x + 16}{(x + 2)^2} = x - 4 + \frac{12(x + 2) - 8}{(x + 2)^2}$

$$= x - 4 + \frac{12}{x + 2} - \frac{8}{(x + 2)^2} = x - 4 + \frac{12}{x + 2} - 8(x + 2)^{-2}$$

$$\int_0^8 \frac{x^3}{x^2 + 4x + 4} dx = \int_0^8 \left(x - 4 + \frac{12}{x + 2} - 8(x + 2)^{-2} \right) dx = \left[\frac{1}{2}x^2 - 4x + 12 \ln|x + 2| + 8(x + 2)^{-1} \right]_0^8$$

$$= \left[\frac{1}{2}x^2 - 4x + 12 \ln|x + 2| + \frac{8}{x + 2} \right]_0^8$$

$$= 32 - 32 + 12 \ln(10) + \frac{4}{5} - (0 - 0 + 12 \ln(2) + 4) = 12 \ln(10) + \frac{4}{5} - 12 \ln(2) - 4$$

$$= 12 \ln(5) - 3\frac{1}{5}$$

c $\frac{4x - 8}{x^2 - 4x - 5} = \frac{4x - 8}{(x + 1)(x - 5)} = \frac{a}{x + 1} + \frac{b}{x - 5} = \frac{a(x - 5) + b(x + 1)}{(x + 1)(x - 5)} = \frac{ax - 5a + bx + b}{(x + 1)(x - 5)}$

$$= \frac{(a + b)x - 5a + b}{(x + 1)(x - 5)}$$

$$\begin{cases} a + b = 4 \\ -5a + b = -8 \end{cases}$$

$$\frac{6a = 12}{a = 2}$$

$$\begin{cases} a = 2 \\ a + b = 4 \end{cases} \Rightarrow \begin{cases} 2 + b = 4 \\ b = 2 \end{cases}$$

$$\int_0^2 \frac{4x - 8}{x^2 - 4x - 5} dx = \int_0^2 \left(\frac{2}{x + 1} + \frac{2}{x - 5} \right) dx = [2 \ln|x + 1| + 2 \ln|x - 5|]_0^2$$

$$= 2 \ln(3) + 2 \ln(3) - (2 \ln(1) + 2 \ln(5)) = 4 \ln(3) - 2 \ln(5)$$

62 a $x^2 + 1 / x^4 \quad + 1 \setminus x^2 - 1$

$$\frac{x^4 + x^2}{-x^2}$$

$$\frac{-x^2 - 1}{2}$$

$$\int_0^1 \frac{x^4 + 1}{x^2 + 1} dx = \int_0^1 \left(x^2 - 1 + \frac{2}{x^2 + 1} \right) dx = \left[\frac{1}{3}x^3 - x + 2 \arctan(x) \right]_0^1$$

$$= \frac{1}{3} - 1 + 2 \arctan(1) - (0 - 0 + 2 \arctan(0)) = -\frac{2}{3} + 2 \cdot \frac{1}{4}\pi = -\frac{2}{3} + \frac{1}{2}\pi$$

b $x^2 + 2x + 1 / x^4 \quad \quad \quad + 1 \setminus x^2 - 2x + 3$

$$\begin{array}{r} x^4 + 2x^3 + x^2 \\ -2x^3 - x^2 \\ \hline -2x^3 - 4x^2 - 2x \\ 3x^2 + 2x \\ \hline 3x^2 + 6x + 3 \\ -4x - 2 \\ \hline \end{array}$$

$$\begin{aligned} \text{Dus } \frac{x^4 + 1}{x^2 + 2x + 1} &= x^2 - 2x + 3 - \frac{4x + 2}{x^2 + 2x + 1} = x^2 - 2x + 3 - \frac{4x + 2}{(x + 1)^2} = x^2 - 2x + 3 - \frac{4(x + 1) - 2}{(x + 1)^2} \\ &= x^2 - 2x + 3 - \frac{4}{x + 1} + \frac{2}{(x + 1)^2} = x^2 - 2x + 3 - \frac{4}{x + 1} + 2(x + 1)^{-2} \end{aligned}$$

$$\begin{aligned} \int_0^1 \frac{x^4 + 1}{x^2 + 2x + 1} dx &= \int_0^1 \left(x^2 - 2x + 3 - \frac{4}{x + 1} + 2(x + 1)^{-2} \right) dx \\ &= \left[\frac{1}{3}x^3 - x^2 + 3x - 4\ln|x + 1| - 2(x + 1)^{-1} \right]_0^1 = \left[\frac{1}{3}x^3 - x^2 + 3x - 4\ln|x + 1| - \frac{2}{x + 1} \right]_0^1 \\ &= \frac{1}{3} - 1 + 3 - 4\ln(2) - 1 - (0 - 0 + 0 - 4\ln(1) - 2) = \frac{1}{3} - 4\ln(2) + 2 = 3\frac{1}{3} - 4\ln(2) \end{aligned}$$

c $x^2 + 4x + 3 / x^4 \quad \quad \quad \setminus x^2 - 4x + 13$

$$\begin{array}{r} x^4 + 4x^3 + 3x^2 \\ -4x^3 - 3x^2 \\ \hline -4x^3 - 16x^2 - 12x \\ 13x^2 + 12x \\ \hline 13x^2 + 52x + 39 \\ -40x - 39 \\ \hline \end{array}$$

$$\begin{aligned} \frac{-40x - 39}{x^2 + 4x + 3} &= \frac{-40x - 39}{(x + 1)(x + 3)} = \frac{a}{x + 1} + \frac{b}{x + 3} = \frac{a(x + 3) + b(x + 1)}{(x + 1)(x + 3)} = \frac{ax + 3a + bx + b}{(x + 1)(x + 3)} \\ &= \frac{(a + b)x + 3a + b}{(x + 1)(x + 3)} \end{aligned}$$

$$\begin{cases} a + b = -40 \\ 3a + b = -39 \\ \hline -2a = -1 \\ a = \frac{1}{2} \end{cases} \quad \left. \begin{matrix} \frac{1}{2} + b = -40 \\ a + b = -40 \end{matrix} \right\} \begin{matrix} b = -40\frac{1}{2} \end{matrix}$$

$$\begin{aligned} \int_0^2 \frac{x^4}{x^2 + 4x + 3} dx &= \int_0^2 \left(x^2 - 4x + 13 + \frac{\frac{1}{2}}{x + 1} - \frac{40\frac{1}{2}}{x + 3} \right) dx \\ &= \left[\frac{1}{3}x^3 - 2x^2 + 13x + \frac{1}{2}\ln|x + 1| - 40\frac{1}{2}\ln|x + 3| \right]_0^2 \\ &= \frac{8}{3} - 8 + 26 + \frac{1}{2}\ln(3) - 40\frac{1}{2}\ln(5) - (0 - 0 + 0 + \frac{1}{2}\ln(1) - 40\frac{1}{2}\ln(3)) \\ &= 20\frac{2}{3} + \frac{1}{2}\ln(3) - 40\frac{1}{2}\ln(5) + 40\frac{1}{2}\ln(3) = 20\frac{2}{3} + 41\ln(3) - 40\frac{1}{2}\ln(5) \end{aligned}$$

63 I $\frac{2x - 5}{x^2 - 5x + 6} = \frac{2x - 5}{(x - 2)(x - 3)} = \frac{a}{x - 2} + \frac{b}{x - 3} = \frac{a(x - 3) + b(x - 2)}{(x - 2)(x - 3)} = \frac{ax - 3a + bx - 2b}{(x - 2)(x - 3)}$

$$= \frac{(a + b)x - 3a - 2b}{(x - 2)(x - 3)}$$

$$\begin{cases} a + b = 2 \\ -3a - 2b = -5 \end{cases} \quad \left| \begin{array}{l} 2 \\ 1 \end{array} \right| \text{ geeft } \begin{cases} 2a + 2b = 4 \\ -3a - 2b = -5 \\ \hline -a = -1 \\ a = 1 \end{cases} \quad \left. \begin{matrix} 1 + b = 2 \\ a + b = 2 \end{matrix} \right\} \begin{matrix} b = 1 \end{matrix}$$

$$f(x) = \frac{1}{x - 2} + \frac{1}{x - 3} \text{ geeft } F(x) = \ln|x - 2| + \ln|x - 3| + c$$

II $F(x) = \int \frac{2x - 5}{x^2 - 5x + 6} dx = \int \frac{1}{x^2 - 5x + 6} d(x^2 - 5x + 6) = \ln|x^2 - 5x + 6| + c$
 $\ln|x - 2| + \ln|x - 3| = \ln|(x - 2)(x - 3)| = \ln|x^2 - 5x + 6|$
 Dus de primitieven komen op hetzelfde neer.

64 a $f(x) = \frac{10x+5}{4x^2-4x+1}$ geeft

$$f'(x) = \frac{(4x^2-4x+1) \cdot 10 - (10x+5) \cdot (8x-4)}{(4x^2-4x+1)^2} = \frac{40x^2-40x+10 - (80x^2-40x+20)}{(2x-1)^4}$$

$$= \frac{40x^2-40x+10-80x^2+40x-20}{(2x-1)^4} = \frac{-40x^2-40x+30}{(2x-1)^4}$$

$$f'(x) = 0 \text{ geeft } -40x^2-40x+30 = 0$$

$$4x^2+4x-3 = 0$$

$$D = 4^2 - 4 \cdot 4 \cdot -3 = 64$$

$$x = \frac{-4 \pm 8}{8} = \frac{1}{2} \vee x = \frac{-4-8}{8} = -1\frac{1}{2}$$

vold. niet

$$\text{min. is } f(-1\frac{1}{2}) = \frac{-15+5}{9+6+1} = \frac{-10}{16} = -\frac{5}{8}$$

$$\text{Dus } B_f = [-\frac{5}{8}, \rightarrow).$$

b $O(V) = \int_1^3 \frac{10x+5}{4x^2-4x+1} dx = \int_1^3 \frac{5(2x-1)+10}{(2x-1)^2} dx = \int_1^3 \left(\frac{5}{2x-1} + \frac{10}{(2x-1)^2} \right) dx$

$$= \int_1^3 \left(\frac{5}{2x-1} + 10(2x-1)^{-2} \right) dx = \left[\frac{5}{2} \ln|2x-1| - 5(2x-1)^{-1} \right]_1^3 = \left[2\frac{1}{2} \ln|2x-1| - \frac{5}{2x-1} \right]_1^3$$

$$= 2\frac{1}{2} \ln(5) - \frac{5}{5} - \left(2\frac{1}{2} \ln(1) - \frac{5}{1} \right) = 2\frac{1}{2} \ln(5) - 1 + 5 = 4 + 2\frac{1}{2} \ln(5)$$

Bladzijde 214

65 a $f(x) = \frac{x^2+4}{x^2+5x+4}$ geeft

$$f'(x) = \frac{(x^2+5x+4) \cdot 2x - (x^2+4) \cdot (2x+5)}{(x^2+5x+4)^2} = \frac{2x^3+10x^2+8x - (2x^3+5x^2+8x+20)}{(x^2+5x+4)^2}$$

$$= \frac{2x^3+10x^2+8x-2x^3-5x^2-8x-20}{(x^2+5x+4)^2} = \frac{5x^2-20}{(x^2+5x+4)^2}$$

$$f'(x) = 0 \text{ geeft } 5x^2-20 = 0$$

$$5x^2 = 20$$

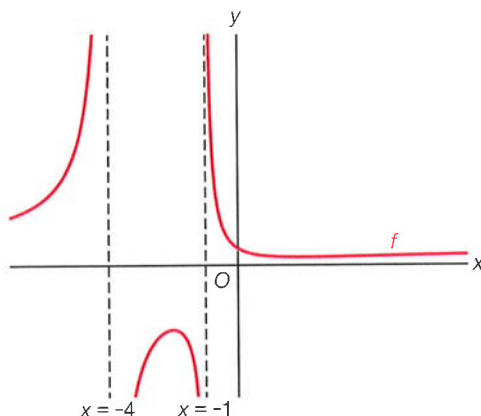
$$x^2 = 4$$

$$x = 2 \vee x = -2$$

$$x^2+5x+4 = 0$$

$$(x+1)(x+4) = 0$$

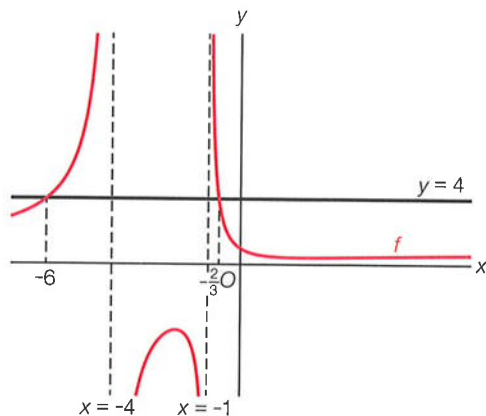
$$x = -1 \vee x = -4$$



$$\text{max. is } f(-2) = -4$$

$$\text{min. is } f(2) = \frac{4}{9}$$

b $f(x) = 4$ geeft $\frac{x^2 + 4}{x^2 + 5x + 4} = 4$
 $4x^2 + 20x + 16 = x^2 + 4$
 $3x^2 + 20x + 12 = 0$
 $D = 20^2 - 4 \cdot 3 \cdot 12 = 256$
 $x = \frac{-20 \pm 16}{6} = -\frac{2}{3} \vee x = \frac{-20 - 16}{6} = -6$



$f(x) \leq 4$ geeft $x \leq -6 \vee -4 < x < -1 \vee x \geq -\frac{2}{3}$

c $x^2 + 5x + 4 / x^2 + 4 \setminus 1$
 $\frac{x^2 + 5x + 4}{x^2 + 5x + 4} -$
 $\frac{-5x}{-5x}$

$$f(x) = 1 - \frac{5x}{x^2 + 5x + 4} = 1 - \frac{5x}{(x+1)(x+4)} = 1 - \left(\frac{a}{x+1} + \frac{b}{x+4} \right) = 1 - \frac{a(x+4) + b(x+1)}{(x+1)(x+4)}$$

$$= 1 - \frac{ax + 4a + bx + b}{(x+1)(x+4)} = 1 - \frac{(a+b)x + 4a + b}{(x+1)(x+4)}$$

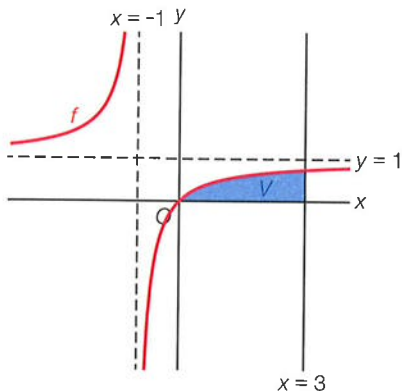
$$\begin{cases} a + b = 5 \\ 4a + b = 0 \end{cases} \quad \begin{array}{l} -3a = 5 \\ a = -1\frac{2}{3} \\ a + b = 5 \end{array} \quad \begin{array}{l} -1\frac{2}{3} + b = 5 \\ b = 6\frac{2}{3} \end{array}$$

$$O(V) = \int_0^6 \left(1 - \frac{5x}{x^2 + 5x + 4} \right) dx = \int_0^6 \left(1 + \frac{1\frac{2}{3}}{x+1} - \frac{6\frac{2}{3}}{x+4} \right) dx = \left[x + 1\frac{2}{3} \ln|x+1| - 6\frac{2}{3} \ln|x+4| \right]_0^6$$

$$= 6 + 1\frac{2}{3} \ln(7) - 6\frac{2}{3} \ln(10) - (0 + 1\frac{2}{3} \ln(1) - 6\frac{2}{3} \ln(4)) = 6 + 1\frac{2}{3} \ln(7) - 6\frac{2}{3} \ln(10) + 6\frac{2}{3} \ln(4)$$

$$= 6 + 1\frac{2}{3} \ln(7) + 6\frac{2}{3} \ln(\frac{2}{5})$$

66 a



$$f(x) = \frac{x}{x+1} = \frac{x+1-1}{x+1} = 1 - \frac{1}{x+1}$$

$$O(V) = \int_0^3 \left(1 - \frac{1}{x+1} \right) dx = \left[x - \ln|x+1| \right]_0^3 = 3 - \ln(4) - (0 - \ln(1)) = 3 - \ln(4)$$

$$\begin{aligned}
 \text{b } I(L) &= \pi \int_0^3 \left(\frac{x}{x+1} \right)^2 dx = \pi \int_0^3 \left(1 - \frac{1}{x+1} \right)^2 dx = \pi \int_0^3 \left(1 - \frac{2}{x+1} + \frac{1}{(x+1)^2} \right) dx \\
 &= \pi \int_0^3 \left(1 - \frac{2}{x+1} + (x+1)^{-2} \right) dx = \pi \left[x - 2 \ln|x+1| - (x+1)^{-1} \right]_0^3 = \pi \left[x - 2 \ln|x+1| - \frac{1}{x+1} \right]_0^3 \\
 &= \pi \left(3 - 2 \ln(4) - \frac{1}{4} - (0 - 2 \ln(1) - 1) \right) = \pi \left(3 - 2 \ln(4) - \frac{1}{4} + 1 \right) = 3\frac{3}{4}\pi - 2\pi \ln(4)
 \end{aligned}$$

67 a $f(x) = \ln(x^2 + 1)$ geeft $f'(x) = \frac{1}{x^2 + 1} \cdot 2x = \frac{2x}{x^2 + 1}$

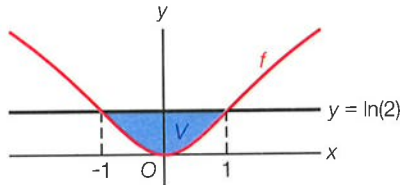
$$\begin{aligned}
 f'(x) &= \frac{3}{5} \text{ geeft } \frac{2x}{x^2 + 1} = \frac{3}{5} \\
 3x^2 + 3 &= 10x \\
 3x^2 - 10x + 3 &= 0 \\
 D &= (-10)^2 - 4 \cdot 3 \cdot 3 = 64 \\
 x &= \frac{10 \pm 8}{6} = 3 \vee x = \frac{10 - 8}{6} = \frac{1}{3}
 \end{aligned}$$

$$f(3) = \ln(10) \text{ en } f\left(\frac{1}{3}\right) = \ln\left(\frac{10}{9}\right)$$

Dus de raakpunten zijn $(3, \ln(10))$ en $(\frac{1}{3}, \ln(\frac{10}{9}))$.

b $f(x) = \ln(2)$ geeft $\ln(x^2 + 1) = \ln(2)$

$$\begin{aligned}
 x^2 + 1 &= 2 \\
 x^2 &= 1 \\
 x &= 1 \vee x = -1
 \end{aligned}$$



$$\begin{aligned}
 \int f(x) dx &= \int \ln(x^2 + 1) dx = x \ln(x^2 + 1) - \int x d \ln(x^2 + 1) = x \ln(x^2 + 1) - \int x \cdot \frac{1}{x^2 + 1} \cdot 2x dx \\
 &= x \ln(x^2 + 1) - \int \frac{2x^2}{x^2 + 1} dx = x \ln(x^2 + 1) - \int \frac{2x^2 + 2 - 2}{x^2 + 1} dx \\
 &= x \ln(x^2 + 1) - \int \left(2 - \frac{2}{x^2 + 1} \right) dx = x \ln(x^2 + 1) - (2x - 2 \arctan(x)) + c \\
 &= x \ln(x^2 + 1) - 2x + 2 \arctan(x) + c \\
 O(V) &= \int_{-1}^1 (\ln(2) - f(x)) dx = [x \ln(2) - (x \ln(x^2 + 1) - 2x + 2 \arctan(x))]_{-1}^1 \\
 &= [x \ln(2) - x \ln(x^2 + 1) + 2x - 2 \arctan(x)]_{-1}^1 \\
 &= \ln(2) - \ln(2) + 2 - 2 \arctan(1) - (-\ln(2) + \ln(2) - 2 - 2 \arctan(-1)) \\
 &= 2 - 2 \cdot \frac{1}{4}\pi + 2 + 2 \cdot \frac{1}{4}\pi = 4 - \pi
 \end{aligned}$$

K.5 Integralen bij parameterkrommen

Bladzijde 216

68 a $\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a) = -(F(a) - F(b)) = -[F(x)]_b^a = -\int_b^a f(x) dx$

b $\int_a^b f(x) dx + \int_b^c f(x) dx = [F(x)]_a^b + [F(x)]_b^c = F(b) - F(a) + F(c) - F(b) = F(c) - F(a)$

$$= [F(x)]_a^c = \int_a^c f(x) dx$$

- 69 a** Bij A hoort $t = 0$, bij B hoort $t = 2$ en bij C hoort $t = 4$, dus

$$O(V) = \int_{x_A}^{x_B} y \, dx - \int_{x_C}^{x_B} y \, dx = \int_{t=0}^{t=2} y \, dx - \int_{t=4}^{t=2} y \, dx.$$

$$\text{Herleiden geeft } O(V) = \int_{t=0}^{t=2} y \, dx - \int_{t=4}^{t=2} y \, dx = \int_{t=0}^{t=2} y \, dx + \int_{t=2}^{t=4} y \, dx = \int_{t=0}^{t=4} y \, dx.$$

$$\begin{aligned} \text{b } O(V) &= \int_{t=0}^{t=4} \left(-\frac{1}{2}t^2 + 2\right) d\left(-\frac{1}{2}t^2 + 2t\right) = \int_0^4 \left(-\frac{1}{2}t^2 + 2\right)(-t + 2) \, dt = \int_0^4 \left(\frac{1}{2}t^3 - t^2 - 2t + 4\right) \, dt \\ &= \left[\frac{1}{8}t^4 - \frac{1}{3}t^3 - t^2 + 4t\right]_0^4 = 32 - \frac{64}{3} - 16 + 16 - 0 = 10\frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{c } O(V) &= \int_{t=4}^{t=0} x \, dy = \int_{t=4}^{t=0} \left(-\frac{1}{2}t^2 + 2t\right) d\left(-\frac{1}{2}t^2 + 2\right) = \int_{t=4}^{t=0} \left(-\frac{1}{2}t^2 + 2t\right) \cdot -t \, dt = \int_{t=4}^{t=0} \left(\frac{1}{2}t^3 - 2t^2\right) \, dt \\ &= \left[\frac{1}{8}t^4 - \frac{2}{3}t^3\right]_4^0 = \frac{1}{8} \cdot 0^4 - \frac{2}{3} \cdot 0^3 - \left(\frac{1}{8} \cdot 4^4 - \frac{2}{3} \cdot 4^3\right) = 0 - 32 + 42\frac{2}{3} = 10\frac{2}{3} \end{aligned}$$

Dus hetzelfde antwoord als bij b.

Bladzijde 217

$$\text{70 a } O(V) = -\int_{t=-2}^{t=2} x \, dy + \int_{t=2}^{t=4} x \, dy = \int_{t=-2}^{t=2} x \, dy + \int_{t=2}^{t=4} x \, dy = \int_{t=-2}^{t=4} x \, dy$$

$$\begin{aligned} \text{b } I(L) &= \pi \int_{t=2}^{t=-2} x^2 \, dy = \pi \int_{t=2}^{t=-2} \left(\frac{1}{2}t^2 - 2\right)^2 d\left(\frac{1}{2}t^2 - 2t\right) = \pi \int_2^{-2} \left(\frac{1}{2}t^2 - 2\right)^2 (t - 2) \, dt \\ &= \pi \int_2^{-2} \left(\frac{1}{4}t^4 - 2t^2 + 4\right)(t - 2) \, dt = \pi \int_2^{-2} \left(\frac{1}{4}t^5 - \frac{1}{2}t^4 - 2t^3 + 4t^2 + 4t - 8\right) \, dt \\ &= \pi \left[\frac{1}{24}t^6 - \frac{1}{10}t^5 - \frac{1}{2}t^4 + 1\frac{1}{3}t^3 + 2t^2 - 8t\right]_2^{-2} \\ &= \pi \left(\frac{64}{24} + \frac{32}{10} - 8 - 10\frac{2}{3} + 8 + 16 - \left(\frac{64}{24} - \frac{32}{10} - 8 + 10\frac{2}{3} + 8 - 16\right)\right) \\ &= \pi \left(2 \cdot \frac{32}{10} - 2 \cdot 10\frac{2}{3} + 2 \cdot 16\right) = 17\frac{1}{15}\pi \end{aligned}$$

Bladzijde 218

$$\begin{aligned} \text{71 } x = 0 \text{ geeft } t^2 - 2t = 0 \\ t(t - 2) = 0 \\ t = 0 \vee t = 2 \end{aligned}$$

$$t = 0 \text{ geeft } (0, 0)$$

$$t = 2 \text{ geeft } (0, 6)$$

$$y = 0 \text{ geeft } t^3 - t = 0$$

$$t(t^2 - 1) = 0$$

$$t = 0 \vee t^2 = 1$$

$$t = 0 \vee t = 1 \vee t = -1$$

$$t = 1 \text{ geeft } (-1, 0)$$

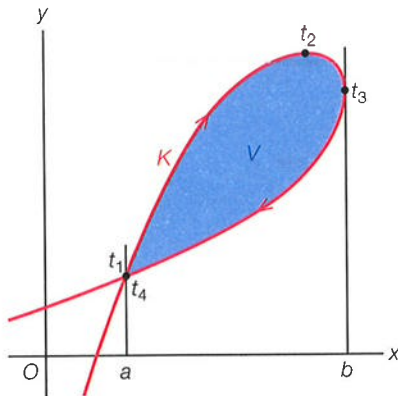
$$t = -1 \text{ geeft } (3, 0)$$

$$\begin{aligned} O(V) &= -\int_{t=1}^{t=0} y \, dx + \int_{t=1}^{t=2} y \, dx = \int_{t=0}^{t=1} y \, dx + \int_{t=1}^{t=2} y \, dx = \int_{t=0}^{t=2} y \, dx = \int_{t=0}^{t=2} (t^3 - t) d(t^2 - 2t) = \int_0^2 (t^3 - t)(2t - 2) \, dt \\ &= \int_0^2 (2t^4 - 2t^3 - 2t^2 + 2t) \, dt = \left[\frac{2}{5}t^5 - \frac{1}{2}t^4 - \frac{2}{3}t^3 + t^2\right]_0^2 = \frac{2}{5} \cdot 32 - \frac{1}{2} \cdot 16 - \frac{2}{3} \cdot 8 + 4 - 0 = 3\frac{7}{15} \end{aligned}$$

$$\begin{aligned} \text{72 } O(V) &= -\int_{t=3}^{t=1} y \, dx + \int_{t=0}^{t=1} y \, dx = \int_{t=1}^{t=3} y \, dx + \int_{t=0}^{t=1} y \, dx = \int_{t=0}^{t=3} y \, dx = \int_0^3 \left(\frac{2}{3}t^2 - 2\frac{2}{3}t + 2\right) d(-t^2 + 2t) \\ &= \int_0^3 \left(\frac{2}{3}t^2 - 2\frac{2}{3}t + 2\right)(-2t + 2) \, dt = \int_0^3 \left(-\frac{4}{3}t^3 + \frac{20}{3}t^2 - \frac{28}{3}t + 4\right) \, dt = \left[-\frac{1}{3}t^4 + \frac{20}{9}t^3 - \frac{14}{3}t^2 + 4t\right]_0^3 \\ &= -27 + 60 - 42 + 12 - 0 = 3 \end{aligned}$$

Bladzijde 219

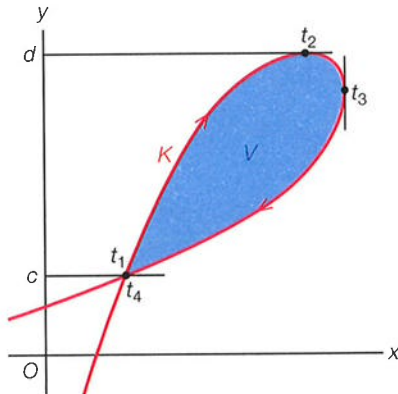
- 73 a** Teken de lijn $x = a$ door het punt waar K zichzelf snijdt en de lijn $x = b$ die K raakt in het punt waarvoor $t = t_3$.



Noem V_1 het vlakdeel ingesloten door de x -as, de lijnen $x = a$ en $x = b$ en het bovenste deel van K en V_2 het vlakdeel ingesloten door de x -as, de lijnen $x = a$ en $x = b$ en het onderste deel van K .

$$\text{Dan geldt } O(V) = O(V_1) - O(V_2) = \int_{t=t_1}^{t=t_3} y \, dx - \int_{t=t_4}^{t=t_3} y \, dx = \int_{t=t_1}^{t=t_3} y \, dx + \int_{t=t_3}^{t=t_4} y \, dx = \int_{t=t_1}^{t=t_4} y \, dx.$$

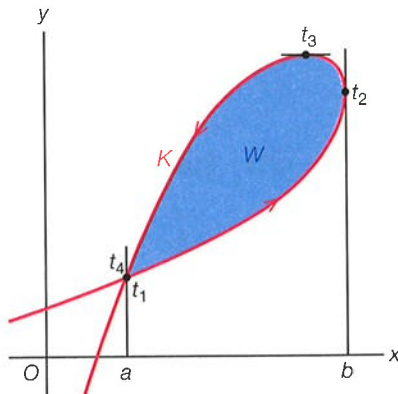
- b** Teken de lijn $y = c$ door het punt waar K zichzelf snijdt en de lijn $y = d$ die K raakt in het punt waarvoor $t = t_2$.



Noem V_3 het vlakdeel ingesloten door de y -as, de lijnen $y = c$ en $y = d$ en het rechterdeel van K en V_4 het vlakdeel ingesloten door de y -as, de lijnen $y = c$ en $y = d$ en het linkerdeel van K .

$$\text{Dan geldt } O(V) = O(V_3) - O(V_4) = \int_{t=t_4}^{t=t_2} x \, dy - \int_{t=t_1}^{t=t_2} x \, dy = \int_{t=t_4}^{t=t_2} x \, dy + \int_{t=t_2}^{t=t_1} x \, dy = \int_{t=t_4}^{t=t_1} x \, dy.$$

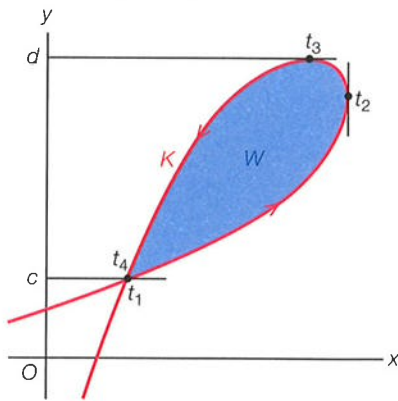
- c** Teken de lijn $x = a$ door het punt waar K zichzelf snijdt en de lijn $x = b$ die K raakt in het punt waarvoor $t = t_2$.



Noem W_1 het vlakdeel ingesloten door de x -as, de lijnen $x = a$ en $x = b$ en het bovenste deel van K en W_2 het vlakdeel ingesloten door de x -as, de lijnen $x = a$ en $x = b$ en het onderste deel van K .

$$\text{Dan geldt } O(W) = O(W_1) - O(W_2) = \int_{t=t_4}^{t=t_2} y \, dx - \int_{t=t_1}^{t=t_2} y \, dx = \int_{t=t_4}^{t=t_2} y \, dx + \int_{t=t_2}^{t=t_1} y \, dx = \int_{t=t_4}^{t=t_1} y \, dx.$$

Teken de lijn $y = c$ door het punt waar K zichzelf snijdt en de lijn $y = d$ die K raakt in het punt waarvoor $t = t_3$.



Noem W_3 het vlakdeel ingesloten door de y -as, de lijnen $y = c$ en $y = d$ en het rechterdeel van K en W_4 het vlakdeel ingesloten door de y -as, de lijnen $y = c$ en $y = d$ en het linkerdeel van K .

$$\text{Dan geldt } O(W) = O(W_3) - O(W_4) = \int_{t=t_1}^{t=t_3} x \, dy - \int_{t=t_4}^{t=t_3} x \, dy = \int_{t=t_1}^{t=t_3} x \, dy + \int_{t=t_3}^{t=t_4} x \, dy = \int_{t=t_1}^{t=t_4} x \, dy.$$

Bladzijde 220

74 a $y = 0$ geeft $4t - t^3 = 0$
 $t(4 - t^2) = 0$
 $t = 0 \vee t^2 = 4$
 $t = 0 \vee t = 2 \vee t = -2$

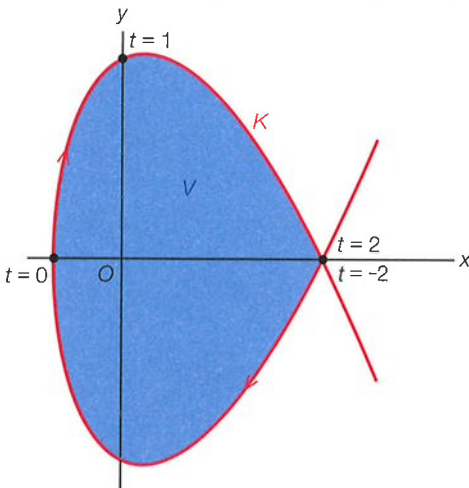
$t = -2$ geeft $(3, 0)$

$t = 0$ geeft $(-1, 0)$

$t = 2$ geeft $(3, 0)$.

$t = 1$ geeft $(0, 3)$

Dus V wordt in negatieve richting omlopen.



$$\begin{aligned} O(V) &= \int_{t=-2}^{t=2} y \, dx = \int_{t=-2}^{t=2} (4t - t^3) d(t^2 - 1) = \int_{-2}^2 (4t - t^3) \cdot 2t \, dt = \int_{-2}^2 (8t^2 - 2t^4) \, dt = \left[2\frac{2}{3}t^3 - \frac{2}{5}t^5 \right]_{-2}^2 \\ &= 2\frac{2}{3} \cdot 2^3 - \frac{2}{5} \cdot 2^5 - \left(2\frac{2}{3} \cdot (-2)^3 - \frac{2}{5} \cdot (-2)^5 \right) = 2\frac{2}{3} \cdot 8 - \frac{2}{5} \cdot 32 - 2\frac{2}{3} \cdot (-8) + \frac{2}{5} \cdot (-32) = 17\frac{1}{15} \end{aligned}$$

b $I(L) = \pi \int_{t=0}^{t=2} y^2 \, dx = \pi \int_{t=0}^{t=2} (4t - t^3)^2 d(t^2 - 1) = \pi \int_0^2 (16t^2 - 8t^4 + t^6) \cdot 2t \, dt$

$$\begin{aligned} &= \pi \int_0^2 (32t^3 - 16t^5 + 2t^7) \, dt = \pi \left[8t^4 - 2\frac{2}{3}t^6 + \frac{1}{4}t^8 \right]_0^2 \\ &= \pi \left(8 \cdot 2^4 - 2\frac{2}{3} \cdot 2^6 + \frac{1}{4} \cdot 2^8 \right) - \pi(0 - 0 + 0) = 21\frac{1}{3}\pi \end{aligned}$$

Bladzijde 221

75 $x = 0$ geeft $-t^2 + 6t = 0$
 $-t(t - 6) = 0$
 $t = 0 \vee t = 6$

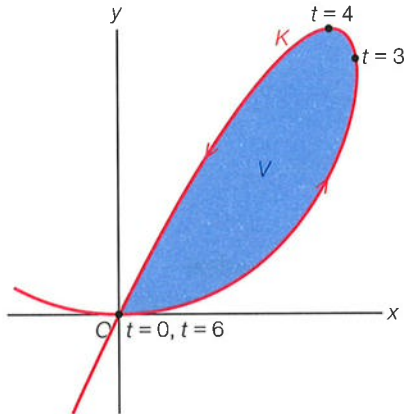
$t = 0$ geeft $(0, 0)$

$t = 6$ geeft $(0, 0)$

$t = 3$ geeft $(9, 9)$

$t = 4$ geeft $(8, 10\frac{2}{3})$

Dus V wordt in positieve richting omlopen.



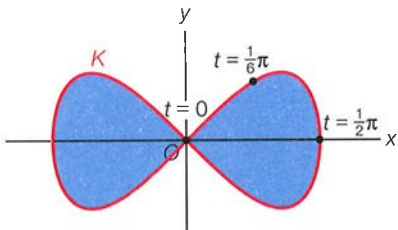
$$\begin{aligned} O(V) &= \int_{t=0}^{t=6} y dx = \int_{t=0}^{t=6} (-\frac{1}{3}t^3 + 2t^2) d(-t^2 + 6t) = \int_{t=0}^0 (-\frac{1}{3}t^3 + 2t^2)(-2t + 6) dt = \int_6^0 (\frac{2}{3}t^4 - 6t^3 + 12t^2) dt \\ &= [\frac{2}{15}t^5 - 1\frac{1}{2}t^4 + 4t^3]_6^0 = 0 - (\frac{2}{15} \cdot 6^5 - 1\frac{1}{2} \cdot 6^4 + 4 \cdot 6^3) = 43\frac{1}{5} \end{aligned}$$

76 a $y = 0$ geeft $\sin(2t) = 0$
 $2t = k \cdot \pi$
 $t = k \cdot \frac{1}{2}\pi$

$t = 0$ geeft $(0, 0)$

$t = \frac{1}{2}\pi$ geeft $(2, 0)$

$t = \frac{1}{6}\pi$ geeft $(1, \frac{1}{2}\sqrt{3})$



Noem V het vlakdeel boven de x -as en rechts van de y -as.

$$\begin{aligned} O(V) &= \int_{t=0}^{t=\frac{1}{2}\pi} y dx = \int_{t=0}^{t=\frac{1}{2}\pi} \sin(2t) d(2\sin(t)) = \int_0^{\frac{1}{2}\pi} \sin(2t) \cdot 2\cos(t) dt = \int_0^{\frac{1}{2}\pi} 2\sin(t)\cos(t) \cdot 2\cos(t) dt \\ &= \int_0^{\frac{1}{2}\pi} 4\sin(t)\cos^2(t) dt = \int_{t=0}^{t=\frac{1}{2}\pi} -4\cos^2(t) d\cos(t) = \int_1^0 -4u^2 du = [-\frac{1}{3}u^3]_1^0 = 0 - (-\frac{1}{3} \cdot 1^3) = \frac{1}{3} \end{aligned}$$

Dus de totale oppervlakte van de vlakdelen is $4 \cdot \frac{1}{3} = \frac{4}{3}$.

b $I(L) = 2 \cdot \pi \int_{t=0}^{t=\frac{1}{2}\pi} y^2 dx = 2\pi \int_{t=0}^{t=\frac{1}{2}\pi} \sin^2(2t) d(2\sin(t)) = 2\pi \int_0^{\frac{1}{2}\pi} (2\sin(t)\cos(t))^2 \cdot 2\cos(t) dt$

$$\begin{aligned} &= 2\pi \int_0^{\frac{1}{2}\pi} 4\sin^2(t)\cos^2(t) \cdot 2\cos(t) dt = 16\pi \int_0^{\frac{1}{2}\pi} \sin^2(t)(1 - \sin^2(t)) d\sin(t) = 16\pi \int_0^1 u^2(1 - u^2) du \\ &= 16\pi \int_0^1 (u^2 - u^4) du = 16\pi [\frac{1}{3}u^3 - \frac{1}{5}u^5]_0^1 = 16\pi(\frac{1}{3} - \frac{1}{5}) = 16\pi(\frac{2}{15}) = \frac{32}{15}\pi \end{aligned}$$

77 a $x = 0$ geeft $2 \sin(t) = 0$

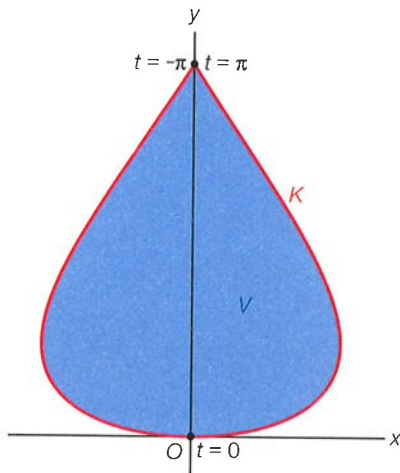
$$t = k \cdot \pi$$

$$t = -\pi \text{ geeft } (0, \frac{1}{2}\pi^2)$$

$$t = 0 \text{ geeft } (0, 0)$$

$$t = \pi \text{ geeft } (0, \frac{1}{2}\pi^2)$$

$$t = \frac{1}{2}\pi \text{ geeft } (2, \frac{1}{8}\pi^2)$$



Dus V wordt in positieve richting omlopen.

K is symmetrisch ten opzichte van de y -as, want

$$x(-t) = 2 \sin(-t) = -2 \sin(t) = -x(t) \text{ en } y(-t) = \frac{1}{2}(-t)^2 = \frac{1}{2}t^2 = y(t).$$

$$O(V) = 2 \cdot \int_{t=0}^{t=\pi} x \, dy = 2 \cdot \int_{t=0}^{t=\pi} 2 \sin(t) \, d\left(\frac{1}{2}t^2\right) = 2 \cdot \int_0^{\pi} 2 \sin(t) \cdot t \, dt = \int_0^{\pi} 4t \sin(t) \, dt$$

$$\int 4t \sin(t) \, dt = \int 4t \, d(-\cos(t)) = 4t \cdot (-\cos(t)) + \int \cos(t) \, d(4t) = -4t \cos(t) + \int 4 \cos(t) \, dt$$

$$= -4t \cos(t) + 4 \sin(t) + c$$

$$\text{Dus } O(V) = [-4t \cos(t) + 4 \sin(t)]_0^{\pi} = -4\pi \cdot -1 + 0 - (0 + 0) = 4\pi.$$

b
$$I(L) = \pi \int_{t=0}^{t=\pi} x^2 \, dy = \pi \int_{t=0}^{t=\pi} 4 \sin^2(t) \, d\left(\frac{1}{2}t^2\right) = \pi \int_0^{\pi} 4 \sin^2(t) \cdot t \, dt = \pi \int_0^{\pi} 4t \sin^2(t) \, dt$$

$$\int 4t \sin^2(t) \, dt = \int 4t \left(\frac{1}{2} - \frac{1}{2} \cos(2t)\right) \, dt = \int (2t - 2t \cos(2t)) \, dt = \int 2t \, dt - \int 2t \cos(2t) \, dt =$$

$$t^2 - \int 2t \, d\left(\frac{1}{2} \sin(2t)\right) = t^2 - 2t \cdot \frac{1}{2} \sin(2t) + \int \frac{1}{2} \sin(2t) \, d(2t) = t^2 - t \sin(2t) + \int \sin(2t) \, dt =$$

$$t^2 - t \sin(2t) - \frac{1}{2} \cos(2t) + c$$

$$\text{Dus } I(L) = \pi \int_0^{\pi} 4t \sin^2(t) \, dt = \pi \left[t^2 - t \sin(2t) - \frac{1}{2} \cos(2t) \right]_0^{\pi} = \pi \left(\pi^2 - 0 - \frac{1}{2} - (0 - 0 - \frac{1}{2}) \right) = \pi^3.$$

Diagnostische toets

Bladzijde 224

1 a
$$F(x) = \int 3x \sqrt{x^2 + 2} \, dx = \int \frac{1}{2} (x^2 + 2)^{\frac{1}{2}} \, d(x^2 + 2) = \int \frac{1}{2} u^{\frac{1}{2}} \, du = \frac{2}{3} \cdot \frac{2}{3} u^{\frac{3}{2}} + c = (x^2 + 2) \sqrt{x^2 + 2} + c$$

b
$$F(x) = \int 3x^2 \cos(x^3 + 2) \, dx = \int \cos(x^3 + 2) \, d(x^3 + 2) = \int \cos(u) \, du = \sin(u) + c = \sin(x^3 + 2) + c$$

c
$$F(x) = \int (4x + 6) \ln(x^2 + 3x) \, dx = \int \ln(x^2 + 3x) \cdot 2 \, d(x^2 + 3x) = \int 2 \ln(u) \, du = 2(u \ln(u) - u) + c$$

$$= 2u \ln(u) - 2u + c = 2(x^2 + 3x) \ln(x^2 + 3x) - 2(x^2 + 3x) + c$$

$$2 \quad a \quad \int_1^e \frac{\sqrt{\ln(x)}}{x} dx = \int_{x=1}^{x=e} \sqrt{\ln(x)} d \ln(x) = \int_0^1 \sqrt{u} du = \int_0^1 u^{\frac{1}{2}} du = \left[\frac{2}{3} u^{\frac{3}{2}} \right]_0^1 = \frac{2}{3} \cdot 1 - \frac{2}{3} \cdot 0 = \frac{2}{3}$$

$$b \quad \int_0^{\frac{1}{2}\pi} \cos(x) \sin^3(x) dx = \int_{x=0}^{x=\frac{1}{2}\pi} \sin^3(x) d \sin(x) = \int_0^1 u^3 du = \left[\frac{1}{4} u^4 \right]_0^1 = \frac{1}{4} \cdot 1 - \frac{1}{4} \cdot 0 = \frac{1}{4}$$

$$c \quad \int_1^3 \frac{6x}{x^2+1} dx = \int_{x=1}^{x=3} \frac{3}{x^2+1} d(x^2+1) = \int_2^{10} \frac{3}{u} du = \left[3 \ln|u| \right]_2^{10} = 3 \ln(10) - 3 \ln(2) = 3 \ln(5)$$

$$3 \quad f(x) = 0 \text{ geeft } -2x \sin(x^2) = 0 \\ x = 0 \vee \sin(x^2) = 0 \\ x = 0 \vee x^2 = k \cdot \pi \\ x = 0 \vee x = \sqrt{k \cdot \pi} \vee x = -\sqrt{k \cdot \pi}$$

$$x \text{ in } [0, \sqrt{2\pi}] \text{ geeft } x = 0 \vee x = \sqrt{\pi} \vee x = \sqrt{2\pi}$$

$$\int -2x \sin(x^2) dx = \int -\sin(x^2) \cdot 2x dx = \int -\sin(x^2) d(x^2) = \int -\sin(u) du = \cos(u) + c = \cos(x^2) + c$$

$$O(V) = \int_0^{\sqrt{\pi}} 2x \sin(x^2) dx = [-\cos(x^2)]_0^{\sqrt{\pi}} = -\cos(\pi) + \cos(0) = 1 + 1 = 2$$

$$O(W) = \int_{\sqrt{\pi}}^{\sqrt{2\pi}} -2x \sin(x^2) dx = [\cos(x^2)]_{\sqrt{\pi}}^{\sqrt{2\pi}} = \cos(2\pi) - \cos(\pi) = 1 + 1 = 2$$

Dus de oppervlakten van de twee vlakdelen zijn gelijk.

$$4 \quad a \quad F(x) = \int 4x \sin(2x) dx = \int -2x \cdot -2 \sin(2x) dx = \int -2x d \cos(2x) = -2x \cos(2x) + \int 2 \cos(2x) dx \\ = -2x \cos(2x) + \sin(2x) + c$$

$$b \quad G(x) = \int x^2 \ln(x) dx = \int \ln(x) d \frac{1}{3} x^3 = \frac{1}{3} x^3 \ln(x) - \int \frac{1}{3} x^3 d \ln(x) = \frac{1}{3} x^3 \ln(x) - \int \frac{1}{3} x^3 \cdot \frac{1}{x} dx \\ = \frac{1}{3} x^3 \ln(x) - \int \frac{1}{3} x^2 dx = \frac{1}{3} x^3 \ln(x) - \frac{1}{9} x^3 + c$$

$$5 \quad a \quad f(x) = (2x - x^2) e^x \text{ geeft } f'(x) = (2 - 2x) \cdot e^x + (2x - x^2) \cdot e^x = (2 - x^2) e^x \\ f'(x) = 0 \text{ geeft } (2 - x^2) e^x = 0 \\ 2 - x^2 = 0 \\ x^2 = 2 \\ x = \sqrt{2} \vee x = -\sqrt{2}$$

$$\text{min. is } f(-\sqrt{2}) = (-2\sqrt{2} - 2) e^{-\sqrt{2}}$$

$$\text{max. is } f(\sqrt{2}) = (2\sqrt{2} - 2) e^{\sqrt{2}}$$

$$b \quad f(x) = 0 \text{ geeft } 2x - x^2 = 0$$

$$x(2 - x) = 0$$

$$x = 0 \vee x = 2$$

$$\int (2x - x^2) e^x dx = \int (2x - x^2) d e^x = (2x - x^2) e^x - \int e^x d(2x - x^2)$$

$$= (2x - x^2) e^x - \int (2 - 2x) e^x dx$$

$$= (2x - x^2) e^x - \int (2 - 2x) d e^x$$

$$= (2x - x^2) e^x - (2 - 2x) e^x + \int e^x d(2 - 2x)$$

$$= (2x - x^2) e^x - (2 - 2x) e^x - \int 2 e^x dx$$

$$= (2x - x^2) e^x - (2 - 2x) e^x - 2 e^x = (-x^2 + 4x - 4) e^x$$

$$O(V) = \int_0^2 f(x) dx = \int_0^2 (2x - x^2) e^x dx = [(-x^2 + 4x - 4) e^x]_0^2 = 0 + 4 = 4$$

$$\begin{aligned}
 \text{6 a } F(x) &= \int 4x^2 \sin(2x) dx = \int 4x^2 d\left(-\frac{1}{2} \cos(2x)\right) = -2x^2 \cos(2x) - \int -\frac{1}{2} \cos(2x) d4x^2 \\
 &= -2x^2 \cos(2x) + \int 4x \cos(2x) dx = -2x^2 \cos(2x) + \int 4x d\left(\frac{1}{2} \sin(2x)\right) \\
 &= -2x^2 \cos(2x) + 2x \sin(2x) - \int \frac{1}{2} \sin(2x) d4x \\
 &= -2x^2 \cos(2x) + 2x \sin(2x) - \int 2 \sin(2x) dx \\
 &= -2x^2 \cos(2x) + 2x \sin(2x) + \cos(2x) + c
 \end{aligned}$$

$$\begin{aligned}
 \text{b } \int e^x \sin(2x) dx &= \int \sin(2x) d e^x = e^x \sin(2x) - \int e^x d \sin(2x) = e^x \sin(2x) - \int 2 \cos(2x) e^x dx \\
 &= e^x \sin(2x) - \int 2 \cos(2x) d e^x = e^x \sin(2x) - 2 e^x \cos(2x) + \int 2 e^x d \cos(2x) \\
 &= e^x \sin(2x) - 2 e^x \cos(2x) - \int 4 e^x \sin(2x) dx
 \end{aligned}$$

$$\text{Dus } \int e^x \sin(2x) dx = e^x \sin(2x) - 2 e^x \cos(2x) - 4 \int e^x \sin(2x) dx$$

$$5 \int e^x \sin(2x) dx = e^x \sin(2x) - 2 e^x \cos(2x)$$

$$\int e^x \sin(2x) dx = \frac{1}{5} e^x \sin(2x) - \frac{2}{5} e^x \cos(2x)$$

$$\text{Dus de primitieven zijn } G(x) = \frac{1}{5} e^x (\sin(2x) - 2 \cos(2x)) + c.$$

$$\begin{aligned}
 \text{7 a } \int_0^{2\sqrt{3}} \frac{4}{x^2 + 4} dx &= \int_0^{2\sqrt{3}} \frac{1}{\frac{1}{4}x^2 + 1} dx = \int_{x=0}^{x=2\sqrt{3}} 2 \cdot \frac{1}{(\frac{1}{2}x)^2 + 1} d\frac{1}{2}x = \left[2 \arctan\left(\frac{1}{2}x\right)\right]_0^{2\sqrt{3}} \\
 &= 2 \arctan(\sqrt{3}) - 2 \arctan(0) = 2 \cdot \frac{1}{3}\pi - 0 = \frac{2}{3}\pi
 \end{aligned}$$

$$\text{b } \int_1^2 \frac{1}{x^2 - 2x + 2} dx = \int_1^2 \frac{1}{(x-1)^2 - 1 + 2} dx = \int_1^2 \frac{1}{(x-1)^2 + 1} dx = \left[\arctan(x-1)\right]_1^2 = \arctan(1) - \arctan(0) = \frac{1}{4}\pi$$

$$\text{c } \int_{-1}^1 \frac{3x^2}{x^6 + 1} dx = \int_{x=-1}^{x=1} \frac{1}{(x^3)^2 + 1} dx^3 = \left[\arctan(x^3)\right]_{-1}^1 = \arctan(1) - \arctan(-1) = \frac{1}{4}\pi - \left(-\frac{1}{4}\pi\right) = \frac{1}{2}\pi$$

$$\text{d } \int_0^1 \frac{\arctan^2(x)}{x^2 + 1} dx = \int_0^1 \arctan^2(x) \cdot \frac{1}{x^2 + 1} dx = \int_{x=0}^{x=1} \arctan^2(x) d \arctan(x) = \int_0^{\frac{1}{4}\pi} u^2 du = \left[\frac{1}{3}u^3\right]_0^{\frac{1}{4}\pi} = \frac{1}{3} \cdot \left(\frac{1}{4}\pi\right)^3 - 0 = \frac{1}{192}\pi^3$$

Bladzijde 225

$$\begin{aligned}
 \text{8 a } f(x) = \frac{1}{2} \text{ geeft } \frac{1}{x^2 - 4x + 5} &= \frac{1}{2} \\
 x^2 - 4x + 5 &= 2 \\
 x^2 - 4x + 3 &= 0 \\
 (x-1)(x-3) &= 0 \\
 x &= 1 \vee x = 3
 \end{aligned}$$

$$\begin{aligned}
 \int_1^3 f(x) dx &= \int_1^3 \frac{1}{x^2 - 4x + 5} dx = \int_1^3 \frac{1}{(x-2)^2 - 4 + 5} dx = \int_1^3 \frac{1}{(x-2)^2 + 1} dx = \left[\arctan(x-2)\right]_1^3 \\
 &= \arctan(1) - \arctan(-1) = \frac{1}{4}\pi + \frac{1}{4}\pi = \frac{1}{2}\pi
 \end{aligned}$$

$$O(V) = \int_1^3 f(x) dx - 2 \cdot \frac{1}{2} = \frac{1}{2}\pi - 1$$

$$\text{b } \int_2^p f(x) dx = \left[\arctan(x-2)\right]_2^p = \arctan(p-2) - \arctan(0) = \arctan(p-2)$$

$$\begin{aligned}
 O(W) &= \frac{1}{3}\pi \text{ geeft } \arctan(p-2) = \frac{1}{3}\pi \\
 p-2 &= \sqrt{3} \\
 p &= 2 + \sqrt{3}
 \end{aligned}$$

$$\text{9 a } F(x) = \int \frac{1}{\sqrt{1-4x^2}} dx = \int \frac{1}{2} \cdot \frac{1}{\sqrt{1-(2x)^2}} d2x = \frac{1}{2} \arcsin(2x) + c$$

$$\text{b } G(x) = \int \frac{5x^4}{\sqrt{1-x^{10}}} dx = \int \frac{1}{\sqrt{1-(x^5)^2}} d(x^5) = \arcsin(x^5) + c$$

$$\begin{aligned}
 \text{10 a } \int_{-1}^1 \frac{3x+4}{x+4} dx &= \int_{-1}^1 \frac{3(x+4)-8}{x+4} dx = \int_{-1}^1 \left(3 - \frac{8}{x+4}\right) dx = [3x - 8 \ln|x+4|]_{-1}^1 \\
 &= 3 - 8 \ln(5) - (-3 - 8 \ln(3)) = 6 - 8 \ln(5) + 8 \ln(3) = 6 + 8 \ln\left(\frac{3}{5}\right)
 \end{aligned}$$

$$\text{b } x+1 / x^3 \quad -x+1 \setminus x^2-x$$

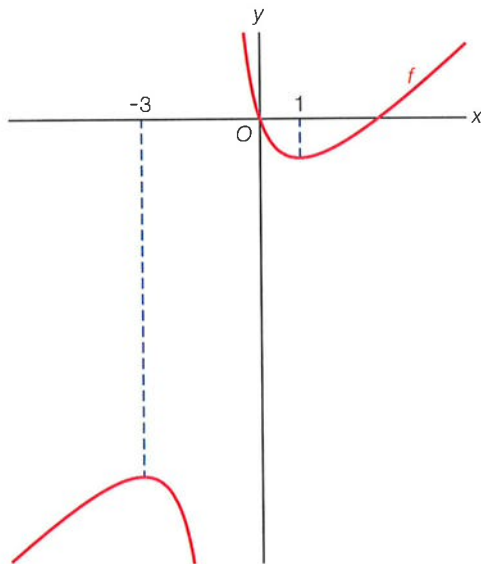
$$\begin{array}{r}
 x^3+x^2 \\
 -x^2-x+1 \\
 \hline
 -x^2-x \\
 \hline
 1
 \end{array}$$

$$\begin{aligned}
 \int_1^3 \frac{x^3-x+1}{x+1} dx &= \int_1^3 \left(x^2-x+\frac{1}{x+1}\right) dx = \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 + \ln|x+1|\right]_1^3 \\
 &= 9 - 4\frac{1}{2} + \ln(4) - \left(\frac{1}{3} - \frac{1}{2} + \ln(2)\right) = 4\frac{2}{3} + \ln(4) - \ln(2) = 4\frac{2}{3} + \ln(2)
 \end{aligned}$$

$$\text{11 a } f(x) = \frac{x^2-3x}{x+1} \text{ geeft}$$

$$f'(x) = \frac{(x+1) \cdot (2x-3) - (x^2-3x) \cdot 1}{(x+1)^2} = \frac{2x^2-3x+2x-3-x^2+3x}{(x+1)^2} = \frac{x^2+2x-3}{(x+1)^2}$$

$$\begin{aligned}
 f'(x) = 0 \text{ geeft } x^2+2x-3 &= 0 \\
 (x-1)(x+3) &= 0 \\
 x = 1 \vee x = -3
 \end{aligned}$$



$$\text{max. is } f(-3) = -9$$

$$\text{min. is } f(1) = -1$$

$$\begin{aligned}
 \text{b } f(x) = 0 \text{ geeft } x^2-3x &= 0 \\
 x(x-3) &= 0 \\
 x = 0 \vee x = 3
 \end{aligned}$$

$$x+1 / x^2-3x \setminus x-4$$

$$\begin{array}{r}
 x^2+x \\
 -4x \\
 \hline
 -4x-4 \\
 \hline
 4
 \end{array}$$

$$\begin{aligned}
 O(V) &= -\int_0^3 f(x) dx = -\int_0^3 \left(x-4+\frac{4}{x+1}\right) dx = -\left[\frac{1}{2}x^2 - 4x + 4 \ln|x+1|\right]_0^3 \\
 &= -(4\frac{1}{2} - 12 + 4 \ln(4)) + (0 - 0 + 4 \ln(1)) = 7\frac{1}{2} - 4 \ln(4)
 \end{aligned}$$

12 a $f(x) = \frac{2x+3}{x^2-6x+10} = \frac{2x-6+9}{x^2-6x+10} = \frac{2x-6}{x^2-6x+10} + \frac{9}{x^2-6x+10}$

$$\int \frac{2x-6}{x^2-6x+10} dx = \int \frac{1}{x^2-6x+10} \cdot (2x-6) dx = \int \frac{1}{x^2-6x+10} d(x^2-6x+10) = \ln|x^2-6x+10| + c$$

$$\int \frac{9}{x^2-6x+10} dx = \int 9 \cdot \frac{1}{(x-3)^2-9+10} dx = \int 9 \cdot \frac{1}{(x-3)^2+1} dx = 9 \arctan(x-3) + c$$

Dus $F(x) = \ln|x^2-6x+10| + 9 \arctan(x-3) + c$

b $x^2 - 4x + 4 / x^2 + 4x \quad \setminus 1$

$$\frac{x^2 - 4x + 4}{8x - 4}$$

$$G(x) = \int \frac{x^2 + 4x}{x^2 - 4x + 4} dx = \int \left(1 + \frac{8x - 4}{x^2 - 4x + 4} \right) dx = \int \left(1 + \frac{8(x-2) + 12}{(x-2)^2} \right) dx$$

$$= \int \left(1 + \frac{8}{x-2} + \frac{12}{(x-2)^2} \right) dx = \int \left(1 + \frac{8}{x-2} + 12(x-2)^{-2} \right) dx$$

$$= x + 8 \ln|x-2| - 12(x-2)^{-1} + c = x + 8 \ln|x-2| - \frac{12}{x-2} + c$$

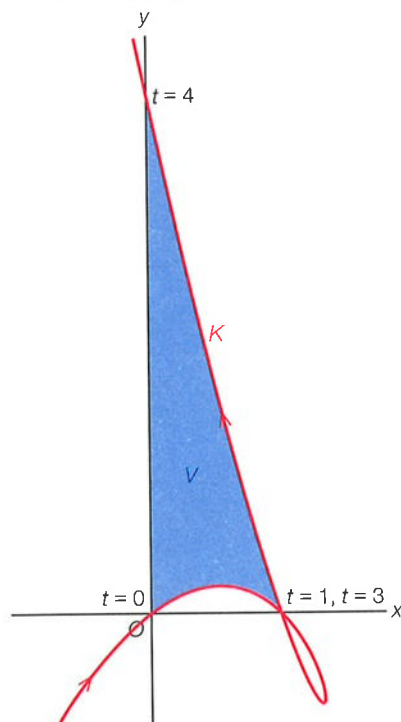
c $h(x) = \frac{6}{x^2-1} = \frac{6}{(x-1)(x+1)} = \frac{a}{x-1} + \frac{b}{x+1} = \frac{a(x+1) + b(x-1)}{(x-1)(x+1)}$

$$= \frac{ax + a + bx - b}{(x-1)(x+1)} = \frac{(a+b)x + a-b}{(x-1)(x+1)}$$

$$\begin{cases} a+b=0 \\ a-b=6 \\ 2a=6 \\ a=3 \\ a+b=0 \end{cases} \Rightarrow b=-3$$

Dus $h(x) = \frac{3}{x-1} - \frac{3}{x+1}$ en dit geeft $H(x) = 3 \ln|x-1| - 3 \ln|x+1| + c$.

- 13 a** $x(t) = 0$ geeft $4t - t^2 = 0$
 $t(4 - t) = 0$
 $t = 0 \vee t = 4$
 $y(t) = 0$ geeft $t^3 - 4t^2 + 3t = 0$
 $t(t^2 - 4t + 3) = 0$
 $t(t - 1)(t - 3) = 0$
 $t = 0 \vee t = 1 \vee t = 3$
 $t = 0$ geeft $(0, 0)$



$$O(V) = \int_{t=4}^{t=3} y \, dx - \int_{t=0}^{t=1} y \, dx$$

$$\int y \, dx = \int (t^3 - 4t^2 + 3t) d(4t - t^2) = \int (t^3 - 4t^2 + 3t)(4 - 2t) \, dt = \int (-2t^4 + 12t^3 - 22t^2 + 12t) \, dt$$

$$= -\frac{2}{5}t^5 + 3t^4 - \frac{22}{3}t^3 + 6t^2 + c$$

$$O(V) = \left[-\frac{2}{5}t^5 + 3t^4 - \frac{22}{3}t^3 + 6t^2 \right]_4^3 - \left[-\frac{2}{5}t^5 + 3t^4 - \frac{22}{3}t^3 + 6t^2 \right]_0^1 = 1\frac{4}{5} - 14\frac{14}{15} - (1\frac{4}{15} - 0) = 15\frac{7}{15}$$

b $O(W) = \int_{t=1}^{t=3} y \, dx = \left[-\frac{2}{5}t^5 + 3t^4 - \frac{22}{3}t^3 + 6t^2 \right]_1^3 = 1\frac{4}{5} - 1\frac{4}{15} = \frac{8}{15}$